CHAPTER 8
Matrices and Determinants

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Section 8.1 Matrices and Systems of Equations

- You should be able to use elementary row operations to produce a row-echelon form (or reduced row-echelon form) of a matrix.
  1. Interchange two rows.
  2. Multiply a row by a nonzero constant.
  3. Add a multiple of one row to another row.
- You should be able to use either Gaussian elimination with back-substitution or Gauss- Jordan elimination to solve a system of linear equations.

Vocabulary Check

1. matrix
2. square
3. main diagonal
4. row; column
5. augmented
6. coefficient
7. row-equivalent
8. reduced row-echelon form
9. Gauss-Jordan elimination

1. Since the matrix has one row and two columns, its order is 1×2.

2. Since the matrix has one row and four columns, its order is 1×4.

3. Since the matrix has three rows and one column, its order is 3×1.

4. Since the matrix has three rows and four columns, its order is 3×4.

5. Since the matrix has two rows and two columns, its order is 2×2.

6. Since the matrix has two rows and three columns, its order is 2×3.

7. $\begin{cases} 4x - 3y = -5 \\ -x + 3y = 12 \end{cases}$

$$\begin{bmatrix} 4 & -3 & \vdots & -5 \\ -1 & 3 & \vdots & 12 \end{bmatrix}$$

8. $\begin{cases} 7x + 4y = 22 \\ 5x - 9y = 15 \end{cases}$

$$\begin{bmatrix} 7 & 4 & \vdots & 22 \\ 5 & -9 & \vdots & 15 \end{bmatrix}$$

9. $\begin{cases} x + 10y - 2z = 2 \\ 5x - 3y + 4z = 0 \\ 2x + y = 6 \end{cases}$

$$\begin{bmatrix} 1 & 10 & -2 & \vdots & 2 \\ 5 & -3 & 4 & \vdots & 0 \\ 2 & 1 & 0 & \vdots & 6 \end{bmatrix}$$

10. $\begin{cases} -x - 8y + 5z = 8 \\ -7x - 15z = -38 \\ 3x - y + 8z = 20 \end{cases}$

$$\begin{bmatrix} -1 & -8 & 5 & \vdots & 8 \\ -7 & 0 & -15 & \vdots & -38 \\ 3 & -1 & 8 & \vdots & 20 \end{bmatrix}$$

11. $\begin{cases} 7x - 5y + z = 13 \\ 19x - 8z = 10 \end{cases}$

$$\begin{bmatrix} 7 & -5 & 1 & \vdots & 13 \\ 19 & 0 & -8 & \vdots & 10 \end{bmatrix}$$

12. $\begin{cases} 9x + 2y - 3z = 20 \\ -25y + 11z = -5 \end{cases}$

$$\begin{bmatrix} 9 & 2 & -3 & \vdots & 20 \\ 0 & -25 & 11 & \vdots & -5 \end{bmatrix}$$

13. $\begin{bmatrix} 1 & 2 & \vdots & 7 \\ 2 & -3 & \vdots & 4 \end{bmatrix}$

$$\begin{cases} x + 2y = 7 \\ 2x - 3y = 4 \end{cases}$$

14. $\begin{bmatrix} 7 & -5 & \vdots & 0 \\ 8 & 3 & \vdots & -2 \end{bmatrix}$

$$\begin{cases} 7x - 5y = 0 \\ 8x + 3y = -2 \end{cases}$$

15. $\begin{bmatrix} 2 & 0 & 5 & \vdots & -12 \\ 0 & 1 & -2 & \vdots & 7 \\ 6 & 3 & 0 & \vdots & 2 \end{bmatrix}$

$$\begin{cases} 2x + 5z = -12 \\ y - 2z = 7 \\ 6x + 3y = 2 \end{cases}$$

16. $\begin{bmatrix} 4 & -5 & -1 & \vdots & 18 \\ -11 & 0 & 6 & \vdots & 25 \\ 3 & 8 & 0 & \vdots & -29 \end{bmatrix}$

$$\begin{cases} 4x - 5y - z = 18 \\ -11x + 6z = 25 \\ 3x + 8y = -29 \end{cases}$$

17. $\begin{bmatrix} 9 & 12 & 3 & 0 & \vdots & 0 \\ -2 & 18 & 5 & 2 & \vdots & 10 \\ 1 & 7 & -8 & 0 & \vdots & -4 \\ 3 & 0 & 2 & 0 & \vdots & -10 \end{bmatrix}$

$$\begin{cases} 9x + 12y + 3z = 0 \\ -2x + 18y + 5z + 2w = 10 \\ x + 7y - 8z = -4 \\ 3x + 2z = -10 \end{cases}$$

18. $\begin{bmatrix} 6 & 2 & -1 & -5 & \vdots & -25 \\ -1 & 0 & 7 & 3 & \vdots & 7 \\ 4 & -1 & -10 & 6 & \vdots & 23 \\ 0 & 8 & 1 & -11 & \vdots & -21 \end{bmatrix}$

$$\begin{cases} 6x + 2y - z - 5w = -25 \\ -x + 7z + 3w = 7 \\ 4x - y - 10z + 6w = 23 \\ 8y + z - 11w = -21 \end{cases}$$

19. $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 10 & 5 \end{bmatrix}$

$\begin{matrix} \\ -2R_1 + R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & 4 & 3 \\ 0 & \boxed{2} & -1 \end{bmatrix}$

20. $\begin{bmatrix} 3 & 6 & 8 \\ 4 & -3 & 6 \end{bmatrix}$

$\begin{matrix} \frac{1}{3}R_1 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & \boxed{2} & \frac{8}{3} \\ 4 & -3 & 6 \end{bmatrix}$

21. $\begin{bmatrix} 1 & 1 & 4 & -1 \\ 3 & 8 & 10 & 3 \\ -2 & 1 & 12 & 6 \end{bmatrix}$

$\begin{matrix} \\ -3R_1 + R_2 \rightarrow \\ 2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 5 & \boxed{-2} & \boxed{6} \\ 0 & 3 & \boxed{20} & \boxed{4} \end{bmatrix}$

$\begin{matrix} \\ \frac{1}{5}R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 1 & 4 & -1 \\ 0 & 1 & -\frac{2}{5} & \frac{6}{5} \\ 0 & 3 & \boxed{20} & \boxed{4} \end{bmatrix}$

22. $\begin{bmatrix} 2 & 4 & 8 & 3 \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{matrix} \frac{1}{2}R_1 \rightarrow \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & \boxed{2} & \boxed{4} & \boxed{\frac{3}{2}} \\ 1 & -1 & -3 & 2 \\ 2 & 6 & 4 & 9 \end{bmatrix}$

$\begin{matrix} \\ -R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 4 & \frac{3}{2} \\ 0 & \boxed{-3} & -7 & \frac{1}{2} \\ 0 & 2 & \boxed{-4} & \boxed{6} \end{bmatrix}$

23. $\begin{bmatrix} -2 & 5 & 1 \\ 3 & -1 & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 13 & 0 & -39 \\ 3 & -1 & -8 \end{bmatrix}$

Add 5 times Row 2 to Row 1.

24. $\begin{bmatrix} 3 & -1 & -4 \\ -4 & 3 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -1 & -4 \\ 5 & 0 & -5 \end{bmatrix}$

Add 3 times Row 1 to Row 2.

25. $\begin{bmatrix} 0 & -1 & -5 & 5 \\ -1 & 3 & -7 & 6 \\ 4 & -5 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 3 & -7 & 6 \\ 0 & -1 & -5 & 5 \\ 0 & 7 & -27 & 27 \end{bmatrix}$

Interchange Row 1 and Row 2. Then add 4 times the new Row 1 to Row 3.

26. $\begin{bmatrix} -1 & -2 & 3 & -2 \\ 2 & -5 & 1 & -7 \\ 5 & 4 & -7 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -2 & 3 & -2 \\ 0 & -9 & 7 & -11 \\ 0 & -6 & 8 & -4 \end{bmatrix}$

Add 2 times Row 1 to Row 2.
Add 5 times Row 1 to Row 3.

27. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & -4 \\ 3 & 1 & -1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 3 & 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -5 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ This matrix is in reduced row-echelon form.

28. $\begin{bmatrix} 7 & 1 \\ 0 & 2 \\ -3 & 4 \\ 4 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} 7 & 1 \\ 0 & 2 \\ -3 & 4 \\ 1 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ -3 & 4 \\ 7 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ 0 & 19 \\ 7 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ 0 & 19 \\ 0 & -34 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 0 & 19 \\ 0 & -34 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ This matrix is in reduced row-echelon form.

29. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

This matrix is in reduced row-echelon form.

30. $\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

This matrix is in reduced row-echelon form.

31. $\begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & -1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

The first nonzero entries in Rows 1 and 2 are not 1. The matrix is not in row-echelon form.

32. $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & 10 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

This matrix is in row-echelon form.

33. $\begin{bmatrix} 1 & 1 & 0 & 5 \\ -2 & -1 & 2 & -10 \\ 3 & 6 & 7 & 14 \end{bmatrix}$

$$\begin{matrix} \\ 2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 7 & -1 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -3R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

34. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 7 & -5 & 14 \\ -2 & -1 & -3 & 8 \end{bmatrix}$

$$\begin{matrix} \\ -3R_1 + R_2 \rightarrow \\ 2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 5 \\ 0 & 3 & -5 & 14 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -3R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

35. $\begin{bmatrix} 1 & -1 & -1 & 1 \\ 5 & -4 & 1 & 8 \\ -6 & 8 & 18 & 0 \end{bmatrix}$

$$\begin{matrix} \\ -5R_1 + R_2 \rightarrow \\ 6R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 6 & 3 \\ 0 & 2 & 12 & 6 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

36. $\begin{bmatrix} 1 & -3 & 0 & -7 \\ -3 & 10 & 1 & 23 \\ 4 & -10 & 2 & -24 \end{bmatrix}$

$$\begin{matrix} \\ 3R_1 + R_2 \rightarrow \\ -4R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -3 & 0 & -7 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & 2 & 4 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -3 & 0 & -7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

37. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 3 & 3 & 3 \\ -1 & 0 & -4 \\ 2 & 4 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

38. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 1 & 3 & 2 \\ 5 & 15 & 9 \\ 2 & 6 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

39. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 1 & 2 & 3 & -5 \\ 1 & 2 & 4 & -9 \\ -2 & -4 & -4 & 3 \\ 4 & 8 & 11 & -14 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

40. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} -2 & 3 & -1 & -2 \\ 4 & -2 & 5 & 8 \\ 1 & 5 & -2 & 0 \\ 3 & 8 & -10 & -30 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

41. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} -3 & 5 & 1 & 12 \\ 1 & -1 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 16 \\ 0 & 1 & 2 & 12 \end{bmatrix}$$

42. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{bmatrix} 5 & 1 & 2 & 4 \\ -1 & 5 & 10 & -32 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & -6 \end{bmatrix}$$

43. $\begin{cases} x - 2y = 4 \\ y = -3 \end{cases}$

$x - 2(-3) = 4$

$x = -2$

Solution: $(-2, -3)$

44. $\begin{cases} x + 5y = 0 \\ y = -1 \end{cases}$

$x + 5(-1) = 0$

$x = 5$

Solution: $(5, -1)$

45. $\begin{cases} x - y + 2z = 4 \\ y - z = 2 \\ z = -2 \end{cases}$

$y - (-2) = 2$

$y = 0$

$x - 0 + 2(-2) = 4$

$x = 8$

Solution: $(8, 0, -2)$

46. $\begin{cases} x + 2y - 2z = -1 \\ y + z = 9 \\ z = -3 \end{cases}$

$y + (-3) = 9$

$y = 12$

$x + 2(12) - 2(-3) = -1$

$x = -31$

Solution: $(-31, 12, -3)$

47. $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -4 \end{array}\right]$

$x = 3$

$y = -4$

Solution: $(3, -4)$

48. $\left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & 10 \end{array}\right]$

$x = -6$
$y = 10$

Solution: $(-6, 10)$

49. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$x = -4$

$y = -10$

$z = 4$

Solution: $(-4, -10, 4)$

50. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$x = 5$
$y = -3$
$z = 0$

Solution: $(5, -3, 0)$

51. $\begin{cases} x + 2y = 7 \\ 2x + y = 8 \end{cases}$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 2 & 1 & 8 \end{array}\right]$$

$$-2R_1 + R_2 \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & -3 & -6 \end{array}\right]$$

$$-\tfrac{1}{3}R_2 \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & 2 \end{array}\right]$$

$\begin{cases} x + 2y = 7 \\ y = 2 \end{cases}$

$y = 2$

$x + 2(2) = 7 \Rightarrow x = 3$

Solution: $(3, 2)$

52. $\begin{cases} 2x + 6y = 16 \\ 2x + 3y = 7 \end{cases}$

$$\left[\begin{array}{rr|r} 2 & 6 & 16 \\ 2 & 3 & 7 \end{array}\right]$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 2 & 6 & 16 \\ 0 & -3 & -9 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{2}R_1 \rightarrow \\ -\frac{1}{3}R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & 3 & 8 \\ 0 & 1 & 3 \end{array}\right]$$

$\begin{cases} x + 3y = 8 \\ \quad\;\; y = 3 \end{cases}$

$y = 3$

$x + 3(3) = 8 \implies x = -1$

Solution: $(-1, 3)$

53. $\begin{cases} 3x - 2y = -27 \\ x + 3y = 13 \end{cases}$

$$\left[\begin{array}{rr|r} 3 & -2 & -27 \\ 1 & 3 & 13 \end{array}\right]$$

$$\begin{array}{r} R_1 \\ R_2 \end{array}\left[\begin{array}{rr|r} 1 & 3 & 13 \\ 3 & -2 & -27 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & 3 & 13 \\ 0 & -11 & -66 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{1}{11}R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & 3 & 13 \\ 0 & 1 & 6 \end{array}\right]$$

$\begin{cases} x + 3y = 13 \\ \quad\;\; y = 6 \end{cases}$

$y = 6$

$x + 3(6) = 13 \implies x = -5$

Solution: $(-5, 6)$

54. $\begin{cases} -x + y = 4 \\ 2x - 4y = -34 \end{cases}$

$$\left[\begin{array}{rr|r} -1 & 1 & 4 \\ 2 & -4 & -34 \end{array}\right]$$

$$\begin{array}{r} (-1)R_1 \rightarrow \\ \left(\frac{1}{2}\right)R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & -1 & -4 \\ 1 & -2 & -17 \end{array}\right]$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & -1 & -4 \\ 0 & -1 & -13 \end{array}\right]$$

$$\begin{array}{r} \\ (-1)R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & -1 & -4 \\ 0 & 1 & 13 \end{array}\right]$$

$\begin{cases} x - y = -4 \\ \quad\; y = 13 \end{cases}$

$y = 13$

$x - 13 = -4 \implies x = 9$

Solution: $(9, 13)$

55. $\begin{cases} -2x + 6y = -22 \\ x + 2y = -9 \end{cases}$

$$\left[\begin{array}{rr|r} -2 & 6 & -22 \\ 1 & 2 & -9 \end{array}\right]$$

$$\begin{array}{r} R_1 \\ R_2 \end{array}\left[\begin{array}{rr|r} 1 & 2 & -9 \\ -2 & 6 & -22 \end{array}\right]$$

$$\begin{array}{r} \\ 2R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & 2 & -9 \\ 0 & 10 & -40 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{10}R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & 2 & -9 \\ 0 & 1 & -4 \end{array}\right]$$

$\begin{cases} x + 2y = -9 \\ \quad\;\; y = -4 \end{cases}$

$y = -4$

$x + 2(-4) = -9 \implies x = -1$

Solution: $(-1, -4)$

56. $\begin{cases} 5x - 5y = -5 \\ -2x - 3y = 7 \end{cases}$

$$\left[\begin{array}{rr|r} 5 & -5 & -5 \\ -2 & -3 & 7 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{5}R_1 \rightarrow \\ \end{array}\left[\begin{array}{rr|r} 1 & -1 & -1 \\ -2 & -3 & 7 \end{array}\right]$$

$$\begin{array}{r} \\ 2R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & -1 & -1 \\ 0 & -5 & 5 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{1}{5}R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} 1 & -1 & -1 \\ 0 & 1 & -1 \end{array}\right]$$

$\begin{cases} x - y = -1 \\ \quad\; y = -1 \end{cases}$

$y = -1$

$x - (-1) = -1 \implies x = -2$

Solution: $(-2, -1)$

57. $\begin{cases} -x + 2y = 1.5 \\ 2x - 4y = 3.0 \end{cases}$

$$\left[\begin{array}{rr|r} -1 & 2 & 1.5 \\ 2 & -4 & 3.0 \end{array}\right]$$

$$\begin{array}{r} \\ 2R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{rr|r} -1 & 2 & 1.5 \\ 0 & 0 & 6.0 \end{array}\right]$$

The system is inconsistent and there is no solution.

58. $\begin{cases} x - 3y = 5 \\ -2x + 6y = -10 \end{cases}$

$$\begin{bmatrix} 1 & -3 & \vdots & 5 \\ -2 & 6 & \vdots & -10 \end{bmatrix}$$

$$2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -3 & \vdots & 5 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

$x - 3y = 5$

$y = a$

$x = 3a + 5$

Solution: $(3a + 5, a)$ where a is a real number

59. $\begin{cases} x \quad\quad - 3z = -2 \\ 3x + y - 2z = 5 \\ 2x + 2y + z = 4 \end{cases}$

$$\begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 3 & 1 & -2 & \vdots & 5 \\ 2 & 2 & 1 & \vdots & 4 \end{bmatrix}$$

$$\begin{matrix} \\ -3R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 2 & 7 & \vdots & 8 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -2R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 0 & -7 & \vdots & -14 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -\frac{1}{7}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 & -3 & \vdots & -2 \\ 0 & 1 & 7 & \vdots & 11 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$\begin{cases} x - 3z = -2 \\ y + 7z = 11 \\ z = 2 \end{cases}$

$z = 2$

$y + 7(2) = 11 \implies y = -3$

$x - 3(2) = -2 \implies x = 4$

Solution: $(4, -3, 2)$

60. $\begin{cases} 2x - y + 3z = 24 \\ 2y - z = 14 \\ 7x - 5y = 6 \end{cases}$

$$\begin{bmatrix} 2 & -1 & 3 & \vdots & 24 \\ 0 & 2 & -1 & \vdots & 14 \\ 7 & -5 & 0 & \vdots & 6 \end{bmatrix}$$

$$R_3 + (-3)R_1 \rightarrow \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 2 & -1 & \vdots & 14 \\ 7 & -5 & 0 & \vdots & 6 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -7R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 2 & -1 & \vdots & 14 \\ 0 & 9 & 63 & \vdots & 468 \end{bmatrix}$$

$$\begin{matrix} \\ 4R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 8 & -4 & \vdots & 56 \\ 0 & 9 & 63 & \vdots & 468 \end{bmatrix}$$

$$\begin{matrix} \\ -R_3 + R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & -1 & -67 & \vdots & -412 \\ 0 & 9 & 63 & \vdots & 468 \end{bmatrix}$$

$$\begin{matrix} \\ \\ 9R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & -1 & -67 & \vdots & -412 \\ 0 & 0 & -540 & \vdots & -3240 \end{bmatrix}$$

$$\begin{matrix} \\ -R_2 \rightarrow \\ -\frac{1}{540}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & -9 & \vdots & -66 \\ 0 & 1 & 67 & \vdots & 412 \\ 0 & 0 & 1 & \vdots & 6 \end{bmatrix}$$

$\begin{cases} x - 2y - 9z = -66 \\ y + 67z = 412 \\ z = 6 \end{cases}$

$z = 6$

$y + 67(6) = 412 \implies y = 10$

$x - 2(10) - 9(6) = -66 \implies x = 8$

Solution: $(8, 10, 6)$

61. $\begin{cases} -x + y - z = -14 \\ 2x - y + z = 21 \\ 3x + 2y + z = 19 \end{cases}$

$$\left[\begin{array}{rrr|r} -1 & 1 & -1 & -14 \\ 2 & -1 & 1 & 21 \\ 3 & 2 & 1 & 19 \end{array}\right]$$

$$\begin{array}{r} -R_1 \rightarrow \\ \\ \\ \end{array} \left[\begin{array}{rrr|r} 1 & -1 & 1 & 14 \\ 2 & -1 & 1 & 21 \\ 3 & 2 & 1 & 19 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & -1 & 1 & 14 \\ 0 & 1 & -1 & -7 \\ 0 & 5 & -2 & -23 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -5R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & -1 & 1 & 14 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & 3 & 12 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{3}R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & -1 & 1 & 14 \\ 0 & 1 & -1 & -7 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$\begin{cases} x - y + z = 14 \\ y - z = -7 \\ z = 4 \end{cases}$$

$z = 4$

$y - 4 = -7 \implies y = -3$

$x - (-3) + 4 = 14 \implies x = 7$

Solution: $(7, -3, 4)$

62. $\begin{cases} 2x + 2y - z = 2 \\ x - 3y + z = -28 \\ -x + y = 14 \end{cases}$

$$\left[\begin{array}{rrr|r} 2 & 2 & -1 & 2 \\ 1 & -3 & 1 & -28 \\ -1 & 1 & 0 & 14 \end{array}\right]$$

$$\begin{array}{r} R_2 \\ R_1 \\ \\ \end{array} \left[\begin{array}{rrr|r} 1 & -3 & 1 & -28 \\ 2 & 2 & -1 & 2 \\ -1 & 1 & 0 & 14 \end{array}\right]$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array} \left[\begin{array}{rrr|r} 1 & -3 & 1 & -28 \\ -1 & 1 & 0 & 14 \\ 2 & 2 & -1 & 2 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & -3 & 1 & -28 \\ 0 & -2 & 1 & -14 \\ 0 & 8 & -3 & 58 \end{array}\right]$$

$$\begin{array}{r} \\ \\ 4R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & -3 & 1 & -28 \\ 0 & -2 & 1 & -14 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{1}{2}R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{rrr|r} 1 & -3 & 1 & -28 \\ 0 & 1 & -\frac{1}{2} & 7 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{cases} x - 3y + z = -28 \\ y - \frac{1}{2}z = 7 \\ z = 2 \end{cases}$$

$z = 2$

$y - \frac{1}{2}(2) = 7 \implies y = 8$

$x - 3(8) + 2 = -28 \implies x = -6$

Solution: $(-6, 8, 2)$

63. $\begin{cases} x + 2y - 3z = -28 \\ 4y + 2z = 0 \\ -x + y - z = -5 \end{cases}$

$$\left[\begin{array}{rrr|r} 1 & 2 & -3 & -28 \\ 0 & 4 & 2 & 0 \\ -1 & 1 & -1 & -5 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{4}R_2 \rightarrow \\ R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & 2 & -3 & -28 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 3 & -4 & -33 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -3R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & 2 & -3 & -28 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{11}{2} & -33 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{2}{11}R_3 \rightarrow \end{array} \left[\begin{array}{rrr|r} 1 & 2 & -3 & -28 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

$$\begin{cases} x + 2y - 3z = -28 \\ y + \frac{1}{2}z = 0 \\ z = 6 \end{cases}$$

$z = 6$

$y + \frac{1}{2}(6) = 0 \implies y = -3$

$x + 2(-3) - 3(6) = -28 \implies x = -4$

Solution: $(-4, -3, 6)$

64. $\begin{cases} 3x - 2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$

$$\left[\begin{array}{rrrcr} 3 & -2 & 1 & \vdots & 15 \\ -1 & 1 & 2 & \vdots & -10 \\ 1 & -1 & -4 & \vdots & 14 \end{array}\right]$$

$$\begin{array}{r} R_3 \\ \\ R_1 \end{array}\left[\begin{array}{rrrcr} 1 & -1 & -4 & \vdots & 14 \\ -1 & 1 & 2 & \vdots & -10 \\ 3 & -2 & 1 & \vdots & 15 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \to \\ -3R_1 + R_3 \to \end{array}\left[\begin{array}{rrrcr} 1 & -1 & -4 & \vdots & 14 \\ 0 & 0 & -2 & \vdots & 4 \\ 0 & 1 & 13 & \vdots & -27 \end{array}\right]$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array}\left[\begin{array}{rrrcr} 1 & -1 & -4 & \vdots & 14 \\ 0 & 1 & 13 & \vdots & -27 \\ 0 & 0 & -2 & \vdots & 4 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{1}{2}R_3 \to \end{array}\left[\begin{array}{rrrcr} 1 & -1 & -4 & \vdots & 14 \\ 0 & 1 & 13 & \vdots & -27 \\ 0 & 0 & 1 & \vdots & -2 \end{array}\right]$$

$\begin{cases} x - y - 4z = 14 \\ y + 13z = -27 \\ z = -2 \end{cases}$

$z = -2$

$y + 13(-2) = -27 \implies y = -1$

$x - (-1) - 4(-2) = 14 \implies x = 5$

Solution: $(5, -1, -2)$

65. $\begin{cases} x + y - 5z = 3 \\ x - 2z = 1 \\ 2x - y - z = 0 \end{cases}$

$$\left[\begin{array}{rrrcr} 1 & 1 & -5 & \vdots & 3 \\ 1 & 0 & -2 & \vdots & 1 \\ 2 & -1 & -1 & \vdots & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -R_1 + R_2 \to \\ -2R_1 + R_3 \to \end{array}\left[\begin{array}{rrrcr} 1 & 1 & -5 & \vdots & 3 \\ 0 & -1 & 3 & \vdots & -2 \\ 0 & -3 & 9 & \vdots & -6 \end{array}\right]$$

$$\begin{array}{r} \\ -R_2 \to \\ \end{array}\left[\begin{array}{rrrcr} 1 & 1 & -5 & \vdots & 3 \\ 0 & 1 & -3 & \vdots & 2 \\ 0 & -3 & 9 & \vdots & -6 \end{array}\right]$$

$$\begin{array}{r} -R_2 + R_1 \to \\ \\ 3R_2 + R_3 \to \end{array}\left[\begin{array}{rrrcr} 1 & 0 & -2 & \vdots & 1 \\ 0 & 1 & -3 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 0 \end{array}\right]$$

$\begin{cases} x - 2z = 1 \\ y - 3z = 2 \end{cases}$

Let $z = a$.

$y - 3a = 2 \implies y = 3a + 2$

$x - 2a = 1 \implies x = 2a + 1$

Solution: $(2a + 1, 3a + 2, a)$ where a is any real number.

66. $\begin{cases} 2x + 3z = 3 \\ 4x - 3y + 7z = 5 \\ 8x - 9y + 15z = 9 \end{cases}$

$$\left[\begin{array}{rrrcr} 2 & 0 & 3 & \vdots & 3 \\ 4 & -3 & 7 & \vdots & 5 \\ 8 & -9 & 15 & \vdots & 9 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1 + R_2 \to \\ -4R_1 + R_3 \to \end{array}\left[\begin{array}{rrrcr} 2 & 0 & 3 & \vdots & 3 \\ 0 & -3 & 1 & \vdots & -1 \\ 0 & -9 & 3 & \vdots & -3 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -3R_2 + R_3 \to \end{array}\left[\begin{array}{rrrcr} 2 & 0 & 3 & \vdots & 3 \\ 0 & -3 & 1 & \vdots & -1 \\ 0 & 0 & 0 & \vdots & 0 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{2}R_1 \to \\ -\frac{1}{3}R_2 \to \\ \end{array}\left[\begin{array}{rrrcr} 1 & 0 & \frac{3}{2} & \vdots & \frac{3}{2} \\ 0 & 1 & -\frac{1}{3} & \vdots & \frac{1}{3} \\ 0 & 0 & 0 & \vdots & 0 \end{array}\right]$$

$z = a$

$y = \frac{1}{3}a + \frac{1}{3}$

$x = -\frac{3}{2}a + \frac{3}{2}$

Solution: $\left(-\frac{3}{2}a + \frac{3}{2}, \frac{1}{3}a + \frac{1}{3}, a\right)$ where a is a real number

67. $\begin{cases} x + 2y + z + 2w = 8 \\ 3x + 7y + 6z + 9w = 26 \end{cases}$

$$\left[\begin{array}{rrrrcr} 1 & 2 & 1 & 2 & \vdots & 8 \\ 3 & 7 & 6 & 9 & \vdots & 26 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_1 + R_2 \to \end{array}\left[\begin{array}{rrrrcr} 1 & 2 & 1 & 2 & \vdots & 8 \\ 0 & 1 & 3 & 3 & \vdots & 2 \end{array}\right]$$

$$\begin{array}{r} -2R_2 + R_1 \to \\ \end{array}\left[\begin{array}{rrrrcr} 1 & 0 & -5 & -4 & \vdots & 4 \\ 0 & 1 & 3 & 3 & \vdots & 2 \end{array}\right]$$

$\begin{cases} x - 5z - 4w = 4 \\ y + 3z + 3w = 2 \end{cases}$

Let $w = a$ and $z = b$.

$y + 3b + 3a = 2 \implies y = 2 - 3b - 3a$

$x - 5b - 4a = 4 \implies x = 4 + 5b + 4a$

Solution: $(4 + 5b + 4a, 2 - 3b - 3a, b, a)$
where a and b are real numbers

68. $\begin{cases} 4x + 12y - 7z - 20w = 22 \\ 3x + 9y - 5z - 28w = 30 \end{cases}$

$$\begin{bmatrix} 4 & 12 & -7 & -20 & \vdots & 22 \\ 3 & 9 & -5 & -28 & \vdots & 30 \end{bmatrix}$$

$$-R_2 + R_1 \longrightarrow \begin{bmatrix} 1 & 3 & -2 & 8 & \vdots & -8 \\ 3 & 9 & -5 & -28 & \vdots & 30 \end{bmatrix}$$

$$\begin{array}{r} \\ -3R_1 + R_2 \longrightarrow \end{array} \begin{bmatrix} 1 & 3 & -2 & 8 & \vdots & -8 \\ 0 & 0 & 1 & -52 & \vdots & 54 \end{bmatrix}$$

$$2R_2 + R_1 \longrightarrow \begin{bmatrix} 1 & 3 & 0 & -96 & \vdots & 100 \\ 0 & 0 & 1 & -52 & \vdots & 54 \end{bmatrix}$$

$w = a$

$z = 52a + 54$

$y = b$

$x = -3b + 96a + 100$

Solution: $(-3b + 96a + 100, b, 52a + 54, a)$ where a and b are real numbers

69. $\begin{cases} -x + y = -22 \\ 3x + 4y = 4 \\ 4x - 8y = 32 \end{cases}$

$$\begin{bmatrix} -1 & 1 & \vdots & -22 \\ 3 & 4 & \vdots & 4 \\ 4 & -8 & \vdots & 32 \end{bmatrix}$$

$$-R_1 \rightarrow \begin{bmatrix} 1 & -1 & \vdots & 22 \\ 3 & 4 & \vdots & 4 \\ 4 & -8 & \vdots & 32 \end{bmatrix}$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -4R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -1 & \vdots & 22 \\ 0 & 7 & \vdots & -62 \\ 0 & -4 & \vdots & -56 \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{7}R_2 \rightarrow \\ -\frac{1}{4}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -1 & \vdots & 22 \\ 0 & 1 & \vdots & -\frac{62}{7} \\ 0 & 1 & \vdots & 14 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -1 & \vdots & 22 \\ 0 & 1 & \vdots & -\frac{62}{7} \\ 0 & 0 & \vdots & \frac{160}{7} \end{bmatrix}$$

The system is inconsistent and there is no solution.

70. $\begin{cases} x + 2y = 0 \\ x + y = 6 \\ 3x - 2y = 8 \end{cases}$

$$\begin{bmatrix} 1 & 2 & \vdots & 0 \\ 1 & 1 & \vdots & 6 \\ 3 & -2 & \vdots & 8 \end{bmatrix}$$

$$\begin{array}{r} \\ -R_1 + R_2 \longrightarrow \\ -3R_1 + R_3 \longrightarrow \end{array} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 6 \\ 0 & -8 & 8 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -8R_2 + R_3 \longrightarrow \end{array} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 6 \\ 0 & 0 & -40 \end{bmatrix}$$

The system in inconsistent and there is no solution.

71. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} 3x + 3y + 12z = 6 \\ x + y + 4z = 2 \\ 2x + 5y + 20z = 10 \\ -x + 2y + 8z = 4 \end{cases} \quad \begin{bmatrix} 3 & 3 & 12 & \vdots & 6 \\ 1 & 1 & 4 & \vdots & 2 \\ 2 & 5 & 20 & \vdots & 10 \\ -1 & 2 & 8 & \vdots & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 4 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix} \Rightarrow \begin{cases} x = 0 \\ y + 4z = 2 \end{cases}$$

Let $z = a$.

$y = 2 - 4a$

$x = 0$

Solution: $(0, 2 - 4a, a)$ where a is any real number

72. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} 2x + 10y + 2z = 6 \\ x + 5y + 2z = 6 \\ x + 5y + z = 3 \\ -3x - 15y - 3z = -9 \end{cases}$$

$$\begin{bmatrix} 2 & 10 & 2 & \vdots & 6 \\ 1 & 5 & 2 & \vdots & 6 \\ 1 & 5 & 1 & \vdots & 3 \\ -3 & -15 & -3 & \vdots & -9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 3 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{cases} z = 3 \\ x + 5y = 0 \end{cases}$$

$z = 3$

$y = a$

$x + 5a = 0 \implies x = -5a$

Solution: $(-5a, a, 3)$ where a is a real number

73. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} 2x + y - z + 2w = -6 \\ 3x + 4y + w = 1 \\ x + 5y + 2z + 6w = -3 \\ 5x + 2y - z - w = 3 \end{cases} \quad \begin{bmatrix} 2 & 1 & -1 & 2 & \vdots & -6 \\ 3 & 4 & 0 & 1 & \vdots & 1 \\ 1 & 5 & 2 & 6 & \vdots & -3 \\ 5 & 2 & -1 & -1 & \vdots & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 1 & 0 & \vdots & 4 \\ 0 & 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

$x = 1$

$y = 0$

$z = 4$

$w = -2$

Solution: $(1, 0, 4, -2)$

74. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} x + 2y + 2z + 4w = 11 \\ 3x + 6y + 5z + 12w = 30 \\ x + 3y - 3z + 2w = -5 \\ 6x - y - z + w = -9 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 2 & 4 & \vdots & 11 \\ 3 & 6 & 5 & 12 & \vdots & 30 \\ 1 & 3 & -3 & 2 & \vdots & -5 \\ 6 & -1 & -1 & 1 & \vdots & -9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & 0 & \vdots & 1 \\ 0 & 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$\begin{cases} x = -1 \\ y = 1 \\ z = 3 \\ w = 1 \end{cases}$$

$w = 1$

$z = 3$

$y = 1$

$x = -1$

Solution: $(-1, 1, 3, 1)$

75. Use the reduced row-echelon form feature of a graphing utility.

$$\begin{cases} x + y + z + w = 0 \\ 2x + 3y + z - 2w = 0 \\ 3x + 5y + z = 0 \end{cases} \qquad \left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & -2 & 0 \\ 3 & 5 & 1 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc:c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{cases} x + 2z = 0 \\ y - z = 0 \\ w = 0 \end{cases}$$

Let $z = a$. Then $x = -2a$ and $y = a$.

Solution: $(-2a, a, a, 0)$ where a is a real number

76. $$\begin{cases} x + 2y + z + 3w = 0 \\ x - y + w = 0 \\ y - z + 2w = 0 \end{cases}$$

$$\left[\begin{array}{cccc:c} 1 & 2 & 1 & 3 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc:c} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$$\begin{cases} x + 2w = 0 \\ y + w = 0 \\ z - w = 0 \end{cases}$$

$w = a, z = a, y = -a, x = -2a$

Solution: $(-2a, -a, a, a)$ where a is a real number

77. (a) $$\begin{cases} x - 2y + z = -6 \\ y - 5z = 16 \\ z = -3 \end{cases}$$

$y - 5(-3) = 16$

$y = 1$

$x - 2(1) + (-3) = -6$

$x = -1$

Solution: $(-1, 1, -3)$

(b) $$\begin{cases} x + y - 2z = 6 \\ y + 3z = -8 \\ z = -3 \end{cases}$$

$y + 3(-3) = -8$

$y = 1$

$x + (1) - 2(-3) = 6$

$x = -1$

Solution: $(-1, 1, -3)$

Both systems yield the same solution, namely $(-1, 1, -3)$.

78. (a) $$\begin{cases} x - 3y + 4z = -11 \\ y - z = -4 \\ z = 2 \end{cases}$$

$y - 2 = -4$

$y = -2$

$x - 3(-2) + 4(2) = -11$

$x = -25$

(b) $$\begin{cases} x + 4y = -11 \\ y + 3z = 4 \\ z = 2 \end{cases}$$

$y + 3(2) = 4$

$y = -2$

$x + 4(-2) = -11$

$x = -3$

The systems do *not* yield the same solution.

79. (a) $\begin{cases} x - 4y + 5z = 27 \\ \quad\;\; y - 7z = -54 \\ \qquad\quad\; z = 8 \end{cases}$

$$y - 7(8) = -54$$
$$y = 2$$
$$x - 4(2) + 5(8) = 27$$
$$x = -5$$

Solution: $(-5, 2, 8)$

(b) $\begin{cases} x - 6y + z = 15 \\ \quad\;\; y + 5z = 42 \\ \qquad\quad\; z = 8 \end{cases}$

$$y + 5(8) = 42$$
$$y = 2$$
$$x - 6(2) + (8) = 15$$
$$x = 19$$

Solution: $(19, 2, 8)$

The systems do *not* yield the same solution.

80. (a) $\begin{cases} x + 3y - z = 19 \\ \quad\;\; y + 6z = -18 \\ \qquad\quad\; z = -4 \end{cases}$

$$y + 6(-4) = -18$$
$$y = 6$$
$$x + 3(6) - (-4) = 19$$
$$x = -3$$

(b) $\begin{cases} x - y + 3z = -15 \\ \quad\;\; y - 2z = 14 \\ \qquad\quad\; z = -4 \end{cases}$

$$y - 2(-4) = 14$$
$$y = 6$$
$$x - 6 + 3(-4) = -15$$
$$x = 3$$

The systems do *not* yield the same solution.

81. $\begin{cases} x + 3y + z = 3 \\ x + 5y + 5z = 1 \\ 2x + 6y + 3z = 8 \end{cases}$

$$\begin{bmatrix} 1 & 3 & 1 & \vdots & 3 \\ 1 & 5 & 5 & \vdots & 1 \\ 2 & 6 & 3 & \vdots & 8 \end{bmatrix}$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 3 & 1 & \vdots & 3 \\ 0 & 2 & 4 & \vdots & -2 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

$$\begin{array}{r} \\ \tfrac{1}{2}R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 3 & 1 & \vdots & 3 \\ 0 & 1 & 2 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

This is a matrix in row-echelon form.

$$\begin{bmatrix} 1 & 3 & \frac{3}{2} & \vdots & 4 \\ 0 & 1 & \frac{7}{4} & \vdots & -\frac{3}{2} \\ 0 & 0 & 1 & \vdots & 2 \end{bmatrix}$$

The row-echelon form feature of a graphing utility yields this form.

There are infinitely many matrices in row-echelon form that correspond to the original system of equations. All such matrices will yield the same solution, namely $(16, -5, 2)$.

82. $\begin{cases} I_1 - I_2 + I_3 = 0 \\ 3I_1 + 4I_2 = 18 \\ \qquad I_2 + 3I_3 = 6 \end{cases}$

$$\begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 3 & 4 & 0 & \vdots & 18 \\ 0 & 1 & 3 & \vdots & 6 \end{bmatrix}$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 0 & 7 & -3 & \vdots & 18 \\ 0 & 1 & 3 & \vdots & 6 \end{bmatrix}$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array} \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 0 & 1 & 3 & \vdots & 6 \\ 0 & 7 & -3 & \vdots & 18 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -7R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 0 & 1 & 3 & \vdots & 6 \\ 0 & 0 & -24 & \vdots & -24 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -\frac{1}{24}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & -1 & 1 & \vdots & 0 \\ 0 & 1 & 3 & \vdots & 6 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$\begin{cases} I_1 - I_2 + I_3 = 0 \\ \qquad I_2 + 3I_3 = 6 \\ \qquad\qquad I_3 = 1 \end{cases}$

$$I_3 = 1$$
$$I_2 + 3(1) = 6 \implies I_2 = 3$$
$$I_1 - 3 + 1 = 0 \implies I_1 = 2$$

83. $\dfrac{4x^2}{(x+1)^2(x-1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$4x^2 = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$

$4x^2 = A(x^2+2x+1) + B(x^2-1) + C(x-1)$

$4x^2 = (A+B)x^2 + (2A+C)x + (A-B-C)$

System of equations: $\begin{aligned} A + B \quad &= 4 \\ 2A \quad + C &= 0 \\ A - B - C &= 0 \end{aligned}$

$$\begin{bmatrix} 1 & 1 & 0 & \vdots & 4 \\ 2 & 0 & 1 & \vdots & 0 \\ 1 & -1 & -1 & \vdots & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

Thus, $A = 1$, $B = 3$, $C = -2$.

So, $\dfrac{4x^2}{(x+1)^2(x-1)} = \dfrac{1}{x-1} + \dfrac{3}{x+1} - \dfrac{2}{(x+1)^2}$.

84. $\dfrac{8x^2}{(x-1)^2(x+1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

$8x^2 = A(x-1)^2 + B(x-1)(x+1) + C(x+1)$

$8x^2 = A(x^2-2x+1) + B(x^2-1) + C(x+1)$

$8x^2 = (A+B)x^2 + (-2A+C)x + (A-B+C)$

System of equations: $\begin{aligned} A + B \quad &= 8 \\ -2A \quad + C &= 0 \\ A - B + C &= 0 \end{aligned}$

$$\begin{bmatrix} 1 & 1 & 0 & \vdots & 8 \\ -2 & 0 & 1 & \vdots & 0 \\ 1 & -1 & 1 & \vdots & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & 6 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix}$$

$A = 2$, $B = 6$, $C = 4$

$\dfrac{8x^2}{(x-1)^2(x+1)} = \dfrac{2}{x+1} + \dfrac{6}{x-1} + \dfrac{4}{(x-1)^2}$

85. $x =$ amount at 7%

$y =$ amount at 8%,

$z =$ amount at 10%

$z = 4x \implies -4x + z = 0$

$$\begin{cases} x + y + z = 1{,}500{,}000 \\ 0.07x + 0.08y + 0.10z = 130{,}500 \\ -4x + z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1{,}500{,}000 \\ 0.07 & 0.08 & 0.10 & 130{,}500 \\ -4 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{matrix} \\ -0.07R_1 + R_2 \rightarrow \\ 4R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 1{,}500{,}000 \\ 0 & 0.01 & 0.03 & \vdots & 25{,}500 \\ 0 & 4 & 5 & \vdots & 6{,}000{,}000 \end{bmatrix}$$

$$\begin{matrix} \\ 100R_2 \rightarrow \\ \\ \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 1{,}500{,}000 \\ 0 & 1 & 3 & \vdots & 2{,}550{,}000 \\ 0 & 4 & 5 & \vdots & 6{,}000{,}000 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -4R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 1{,}500{,}000 \\ 0 & 1 & 3 & \vdots & 2{,}550{,}000 \\ 0 & 0 & -7 & \vdots & -4{,}200{,}000 \end{bmatrix}$$

$$\begin{matrix} \\ \\ -\frac{1}{7}R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & 1 & 1 & \vdots & 1{,}500{,}000 \\ 0 & 1 & 3 & \vdots & 2{,}550{,}000 \\ 0 & 0 & 1 & \vdots & 600{,}000 \end{bmatrix}$$

$$\begin{cases} x + y + z = 1{,}500{,}000 \\ y + 3z = 2{,}550{,}000 \\ z = 600{,}000 \end{cases}$$

$y + 3(600{,}000) = 2{,}550{,}000 \implies y = 750{,}000$

$x + 750{,}000 + 600{,}000 = 1{,}500{,}000 \implies x = 150{,}000$

Solution: \$150,000 at 7%, \$750,000 at 8%, and \$600,000 at 10%

86. $x =$ amount at 9%, $y =$ amount at 10%, $z =$ amount at 12%

$$\begin{aligned} x + y + z &= 500{,}000 \\ 0.09x + 0.10y + 0.12z &= 52{,}000 \\ 2.5x - y &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 500{,}000 \\ 0.09 & 0.10 & 0.12 & \vdots & 52{,}000 \\ 2.5 & -1 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ -0.09R_1 + R_2 \rightarrow \\ -2.5R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 500{,}000 \\ 0 & 0.01 & 0.03 & \vdots & 7{,}000 \\ 0 & -3.5 & -2.5 & \vdots & -1{,}250{,}000 \end{bmatrix}$$

$$\begin{array}{r} \\ 100R_2 \rightarrow \\ 2R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 500{,}000 \\ 0 & 1 & 3 & \vdots & 700{,}000 \\ 0 & -7 & -5 & \vdots & -2{,}500{,}000 \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ 7R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -2 & \vdots & -200{,}000 \\ 0 & 1 & 3 & \vdots & 700{,}000 \\ 0 & 0 & 16 & \vdots & 2{,}400{,}000 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ \frac{1}{16}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & -2 & \vdots & -200{,}000 \\ 0 & 1 & 3 & \vdots & 700{,}000 \\ 0 & 0 & 1 & \vdots & 150{,}000 \end{bmatrix}$$

$$\begin{cases} x - 2z = -200{,}000 \\ y + 3z = 700{,}000 \\ z = 150{,}000 \end{cases}$$

$y + 3(150{,}000) = 700{,}000 \implies y = 250{,}000$

$x - 2(150{,}000) = -200{,}000 \implies x = 100{,}000$

Solution: $(100{,}000, 250{,}000, 150{,}000)$

Answer: \$100,000 at 9%, \$250,000 at 10%, \$150,000 at 12%

87. $y = ax^2 + bx + c$

$$\begin{cases} a + b + c = 8 \\ 4a + 2b + c = 13 \\ 9a + 3b + c = 20 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 8 \\ 4 & 2 & 1 & \vdots & 13 \\ 9 & 3 & 1 & \vdots & 20 \end{bmatrix}$$

$$\begin{array}{r} \\ -4R_1 + R_2 \rightarrow \\ -9R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 8 \\ 0 & -2 & -3 & \vdots & -19 \\ 0 & -6 & -8 & \vdots & -52 \end{bmatrix}$$

$$\begin{array}{r} \\ -\frac{1}{2}R_2 \rightarrow \\ -3R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 8 \\ 0 & 1 & \frac{3}{2} & \vdots & \frac{19}{2} \\ 0 & 0 & 1 & \vdots & 5 \end{bmatrix}$$

$$\begin{cases} a + b + c = 8 \\ b + \frac{3}{2}c = \frac{19}{2} \\ c = 5 \end{cases}$$

$c = 5$

$b + \frac{3}{2}(5) = \frac{19}{2} \implies b = 2$

$a + 2 + 5 = 8 \implies a = 1$

Equation of parabola: $y = x^2 + 2x + 5$

88. $f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 9$

$f(2) = 4a + 2b + c = 8$

$f(3) = 9a + 3b + c = 5$

$$\begin{bmatrix} 1 & 1 & 1 & \vdots & 9 \\ 4 & 2 & 1 & \vdots & 8 \\ 9 & 3 & 1 & \vdots & 5 \end{bmatrix}$$

$$\begin{array}{r} \\ -4R_1 + R_2 \rightarrow \\ -9R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 9 \\ 0 & -2 & -3 & \vdots & -28 \\ 0 & -6 & -8 & \vdots & -76 \end{bmatrix}$$

$$\begin{array}{r} \\ -\frac{1}{2}R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 9 \\ 0 & 1 & \frac{3}{2} & \vdots & 14 \\ 0 & -6 & -8 & \vdots & -76 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ 6R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 1 & \vdots & 9 \\ 0 & 1 & \frac{3}{2} & \vdots & 14 \\ 0 & 0 & 1 & \vdots & 8 \end{bmatrix}$$

$$\begin{cases} a + b + c = 9 \\ b + \frac{3}{2}c = 14 \\ c = 8 \end{cases}$$

$c = 8$

$b + \frac{3}{2}(8) = 14 \implies b = 2$

$a + (2) + (8) = 9 \implies a = -1$

Equation of parabola: $y = -x^2 + 2x + 8$

89. (a) $(0, 5.0),\ (15, 9.6),\ (30, 12.4)$

$y = ax^2 + bx + c$

$$\begin{cases} c = 5 \\ 225a + 15b + c = 9.6 \implies 225a + 15b = 4.6 \\ 900a + 30b + c = 12.4 \implies 900a + 30b = 7.4 \end{cases}$$

$$\begin{bmatrix} 225 & 15 & \vdots & 4.6 \\ 900 & 30 & \vdots & 7.4 \end{bmatrix}$$

$$-4R_1 + R_2 \rightarrow \begin{bmatrix} 225 & 15 & \vdots & 4.6 \\ 0 & -30 & \vdots & -11 \end{bmatrix}$$

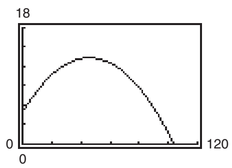
$$\begin{matrix} \frac{1}{225}R_1 \rightarrow \\ \left(-\frac{1}{30}\right)R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & \frac{1}{15} & \vdots & \frac{23}{1125} \\ 0 & 1 & \vdots & \frac{11}{30} \end{bmatrix}$$

$$\begin{cases} a + \frac{1}{15}b = \frac{23}{1125} \\ b = \frac{11}{30} \end{cases}$$

$$a + \frac{1}{15}\left(\frac{11}{30}\right) = \frac{23}{1125} \implies a = -\frac{1}{250} = -0.004$$

Equation of parabola: $y = -0.004x^2 + 0.367x + 5.$

(b)



(c) The maximum height is approximately 13 feet and the ball strikes the ground at approximately 104 feet.

(d) The maximum occurs at the vertex.

$$x = -\frac{b}{2a} = \frac{-0.367}{2(-0.004)} = 45.875$$

$$y = -0.004(45.875)^2 + 0.367(45.875) + 5$$

$$= 13.418 \text{ feet}$$

The ball strikes the ground when $y = 0$.

$$-0.004x^2 + 0.367x + 5 = 0$$

By the Quadratic Formula and using the positive value for x we have $x \approx 103.793$ feet.

(e) The values found in part (d) are more accurate, but still very close to the estimates found in part (c).

90. (a) $f(x) = at^2 + bt + c$

$f(7) = 49a + 7b + c = 2.8$

$f(9) = 81a + 9b + c = 3.3$

$f(11) = 121a + 11b + c = 5.3$

$$\begin{cases} 49a + 7b + c = 2.8 \\ 81a + 9b + c = 3.3 \\ 121a + 11b + c = 5.3 \end{cases}$$

$$\begin{bmatrix} 49 & 7 & 1 & 2.8 \\ 81 & 9 & 1 & 3.3 \\ 121 & 11 & 1 & 5.3 \end{bmatrix}$$

$$\frac{1}{49}R_1 \rightarrow \begin{bmatrix} 1 & \frac{1}{7} & \frac{1}{49} & \frac{2}{35} \\ 81 & 9 & 1 & 3.3 \\ 121 & 11 & 1 & 5.3 \end{bmatrix}$$

$$\begin{matrix} \\ -81R_1 + R_2 \rightarrow \\ -121R_1 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & \frac{1}{7} & \frac{1}{49} & \frac{2}{35} \\ 0 & -\frac{18}{7} & -\frac{32}{49} & -\frac{93}{70} \\ 0 & -\frac{44}{7} & -\frac{72}{49} & -\frac{113}{70} \end{bmatrix}$$

$$-\frac{7}{18}R_2 \rightarrow \begin{bmatrix} 1 & \frac{1}{7} & \frac{1}{49} & \frac{2}{35} \\ 0 & 1 & \frac{16}{63} & \frac{31}{60} \\ 0 & -\frac{44}{7} & -\frac{72}{49} & -\frac{113}{70} \end{bmatrix}$$

$$\begin{matrix} \\ \\ \frac{44}{7}R_2 + R_3 \rightarrow \end{matrix} \begin{bmatrix} 1 & \frac{1}{7} & \frac{1}{49} & \frac{2}{35} \\ 0 & 1 & \frac{16}{63} & \frac{31}{60} \\ 0 & 0 & \frac{8}{63} & \frac{49}{30} \end{bmatrix}$$

$$a + \frac{1}{7}b + \frac{1}{49}c = \frac{2}{35}$$

$$b + \frac{16}{63}c = \frac{31}{60}$$

$$\frac{8}{63}c = \frac{49}{30}$$

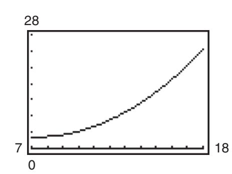
$$c = \frac{49}{30} \cdot \frac{63}{8} = \frac{1029}{80} = 12.86$$

$$b + \frac{16}{63}(12.86) = \frac{31}{60} \implies b = -2.75$$

$$a + \frac{1}{7}(-2.75) + \frac{1}{49}(12.86) = \frac{2}{35} \implies a = 0.1875$$

Equation of parabola: $y = 0.1875t^2 - 2.75t + 12.86$

(b)



(c) For 2003, $t = 13$.

$$y = 0.1875(13^2) - 2.75(13) + 12.86 = 8.8$$

When compared to the actual value of 6.3, this is not very accurate.

(d) For 2008, $t = 18$.

$$y = 0.1875(18^2) - 2.75(18) + 12.86 = 24.11$$

The model estimates that in 2008, 24.11 million people will participate in snowboarding. This indicates that the number of participants will almost triple in 5 years which is probably not a reasonable estimate.

91. (a) $x_1 + x_3 = 600$

$x_1 = x_2 + x_4 \implies x_1 - x_2 - x_4 = 0$

$x_2 + x_5 = 500$

$x_3 + x_6 = 600$

$x_4 + x_7 = x_6 \implies x_4 - x_6 + x_7 = 0$

$x_5 + x_7 = 500$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 600 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 600 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 500 \end{array}\right]$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \\ R_2 + R_3 \rightarrow \\ R_3 + R_4 \rightarrow \\ R_4 + R_5 \rightarrow \\ -R_5 + R_6 \rightarrow \end{array}\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 600 \\ 0 & -1 & -1 & -1 & 0 & 0 & 0 & -600 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 & -100 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 500 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -R_3 + R_2 \rightarrow \\ -R_4 + R_3 \rightarrow \\ -R_4 \rightarrow \\ \\ \\ \end{array}\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 600 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & -500 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & -600 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & -500 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -R_2 \rightarrow \\ -R_3 \rightarrow \\ \\ \\ \\ \end{array}\left[\begin{array}{ccccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 600 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 500 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 600 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & -500 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + x_3 = 600 \\ x_2 + x_5 = 500 \\ x_3 + x_6 = 600 \\ x_4 - x_5 - x_6 = -500 \\ x_5 + x_7 = 500 \end{cases}$$

Let $x_7 = t$ and $x_6 = s$, then $x_5 = 500 - t$,
$x_4 = -500 + s + (500 - t) = s - t$,
$x_3 = 600 - s, x_2 = 500 - (500 - t) = t$,
$x_1 = 600 - (600 - s) = s$.

Solution: $(s, t, 600 - s, s - t, 500 - t, s, t)$

(b) $s = 0, t = 0$: $x_1 = 0, x_2 = 0, x_3 = 600, x_4 = 0, x_5 = 500, x_6 = 0, x_7 = 0$

(c) $s = 0, t = -500$: $x_1 = 0, x_2 = -500, x_3 = 600, x_4 = 500, x_5 = 1000, x_6 = 0, x_7 = -500$

92. (a) $x_1 + x_2 = 300$

$x_1 + x_3 = 150 + x_4 \implies x_1 + x_3 - x_4 = 150$

$x_2 + 200 = x_3 + x_5 \implies x_2 - x_3 - x_5 = -200$

$x_4 + x_5 = 350$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \vdots & 300 \\ 1 & 0 & 1 & -1 & 0 & \vdots & 150 \\ 0 & 1 & -1 & 0 & -1 & \vdots & -200 \\ 0 & 0 & 0 & 1 & 1 & \vdots & 350 \end{bmatrix}$$

$$\begin{array}{l} \\ -R_1 + R_2 \rightarrow \\ \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \vdots & 300 \\ 0 & -1 & 1 & -1 & 0 & \vdots & -150 \\ 0 & 1 & -1 & 0 & -1 & \vdots & -200 \\ 0 & 0 & 0 & 1 & 1 & \vdots & 350 \end{bmatrix}$$

$$\begin{array}{l} \\ \\ R_2 + R_3 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \vdots & 300 \\ 0 & -1 & 1 & -1 & 0 & \vdots & -150 \\ 0 & 0 & 0 & -1 & -1 & \vdots & -350 \\ 0 & 0 & 0 & 1 & 1 & \vdots & 350 \end{bmatrix}$$

$$\begin{array}{r} \\ -R_2 \rightarrow \\ -R_3 \rightarrow \\ R_3 + R_4 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \vdots & 300 \\ 0 & 1 & -1 & 1 & 0 & \vdots & 150 \\ 0 & 0 & 0 & 1 & 1 & \vdots & 350 \\ 0 & 0 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{cases} x_1 + x_2 = 300 \\ x_2 - x_3 + x_4 = 150 \\ x_4 + x_5 = 350 \end{cases}$$

Let $x_5 = t$.

$x_4 + t = 350 \implies x_4 = 350 - t$

Let $x_3 = s$.

$x_2 - s + (350 - t) = 150 \implies x_2 = -200 + s + t$

$x_1 + (-200 + s + t) = 300 \implies x_1 = 500 - s - t$

Solution: $x_1 = 500 - s - t, x_2 = -200 + s + t, x_3 = s, x_4 = 350 - t, x_5 = t$, where s and t are real numbers.

(b) When $x_2 = 200$ and $x_3 = 50$,

$x_2 = -200 + s + t$

$200 = -200 + 50 + t \implies t = 350.$

$x_1 = 100, x_2 = 200, x_3 = 50, x_4 = 0, x_5 = 350$

(c) When $x_2 = 150$ and $x_3 = 0$,

$x_2 = -200 + s + t$

$150 = -200 + 0 + t \implies t = 350.$

$x_1 = 150, x_2 = 150, x_3 = 0, x_4 = 0, x_5 = 350$

93. False. It is a 2×4 matrix.

94. False. The rows are in the wrong order. To change this matrix to reduced row-echelon form, interchange Row 1 and Row 4, and interchange Row 2 and Row 3.

95. False. Gaussian elimination reduces a matrix until a row-echelon form is obtained and Gauss-Jordan elimination reduces a matrix until a reduced row-echelon form is obtained.

96. $z = a$

$y = -4a + 1$

$x = -3a - 2$

One possible system is:

$$\begin{cases} x + y + 7z = (-3a - 2) + (-4a + 1) + 7a = -1 \\ x + 2y + 11z = (-3a - 2) + 2(-4a + 1) + 11a = 0 \\ 2x + y + 10z = 2(-3a - 2) + (-4a + 1) + 10a = -3 \end{cases} \quad \text{or} \quad \begin{cases} x + y + 7z = -1 \\ x + 2y + 11z = 0 \\ 2x + y + 10z = -3 \end{cases}$$

(Note that the coefficients of x, y, and z have been chosen so that the a-terms cancel.)

97. (a) In the row-echelon form of an augmented matrix that corresponds to an inconsistent system of linear equations, there exists a row consisting of all zeros except for the entry in the last column.

(b) In the row-echelon form of an augmented matrix that corresponds to a system with an infinite number of solutions, there are fewer rows with nonzero entries than there are variables and no row has the first non-zero value in the last column.

98. 1. Interchange two rows.

2. Multiply a row by a nonzero constant.

3. Add a multiple of one row to another row.

99. They are the same.

100. A matrix in row-echelon form is in reduced row-echelon form if every column that has a leading 1 has zeros in every position above and below its leading 1.

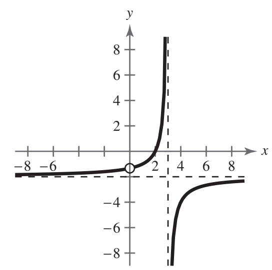
101. $f(x) = \dfrac{2x^2 - 4x}{3x - x^2} = \dfrac{2x - 4}{3 - x}, x \neq 0$

x	-2	-1	0	1	2	3	4	5
$f(x)$	-1.6	-1.5	undef.	-1	0	undef.	-4	-3

Vertical asymptote: $x = 3$

Horizontal asymptote: $y = -2$

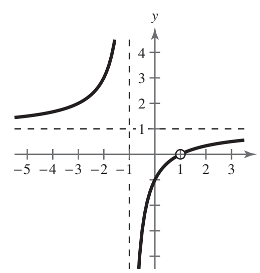
Intercept: $(2, 0)$



102. $f(x) = \dfrac{x^2 - 2x + 1}{x^2 - 1} = \dfrac{\cancel{(x-1)}(x-1)}{\cancel{(x-1)}(x+1)} = \dfrac{x-1}{x+1}$

The graph has a vertical asymptote at $x = -1$ and a discontinuity at $x = 1$.

Since the degrees of the numerator and the denominator are the same, there is a horizontal asymptote at $y = 1$.

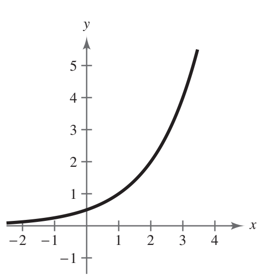


103. $f(x) = 2^{x-1}$

x	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

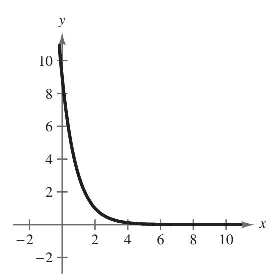
Horizontal asymptote: $y = 0$

Intercept: $\left(0, \frac{1}{2}\right)$



104. $g(x) = 3^{-x+2}$

x	-1	0	1	2	3	4
y	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

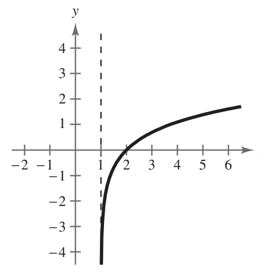


105. $h(x) = \ln(x - 1)$

x	1.5	2	3	4	5
$h(x)$	-0.693	0	0.693	1.099	1.386

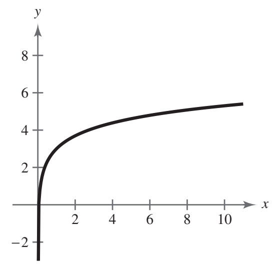
Vertical asymptote: $x = 1$

Intercept: $(2, 0)$



106. $f(x) = 3 + \ln x \implies y - 3 = \ln x \implies e^{y-3} = x$

x	0.05	0.14	0.37	1	2.72
y	0	1	2	3	4



Section 8.2 Operations with Matrices

- $A = B$ if and only if they have the same order and $a_{ij} = b_{ij}$.
- You should be able to perform the operations of matrix addition, scalar multiplication, and matrix multiplication.
- Some properties of matrix addition and scalar multiplication are:
  (a) $A + B = B + A$
  (b) $A + (B + C) = (A + B) + C$
  (c) $(cd)A = c(dA)$
  (d) $1A = A$
  (e) $c(A + B) = cA + cB$
  (f) $(c + d)A = cA + dA$
- You should remember that $AB \neq BA$ in general.
- Some properties of matrix multiplication are:
  (a) $A(BC) = (AB)C$
  (b) $A(B + C) = AB + AC$
  (c) $(A + B)C = AC + BC$
  (d) $c(AB) = (cA)B = A(cB)$
- You should know that I_n, the identity matrix of order n, is an $n \times n$ matrix consisting of ones on its main diagonal and zeros elsewhere. If A is an $n \times n$ matrix, then $AI_n = I_nA = A$.

Vocabulary Check

1. equal

2. scalars

3. zero; O

4. identity

5. (a) (iii) (b) (iv) (c) (i) (d) (v) (e) (ii)

6. (a) (ii) (b) (iv) (c) (i) (d) (iii)

1. $x = -4,\ y = 22$

2. $x = 13, y = 12$

3. $2x + 1 = 5,\ 3x = 6,\ 3y - 5 = 4$

$x = 2,\ y = 3$

4. $x + 2 = 2x + 6 \qquad 2y = 18$

$-4 = x \qquad y = 9$

$2x = -8 \qquad y + 2 = 11$

$x = -4 \qquad y = 9$

5. (a) $A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1+2 & -1-1 \\ 2-1 & -1+8 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & 7 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1-2 & -1+1 \\ 2+1 & -1-8 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 3 & -9 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(-1) \\ 3(2) & 3(-1) \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ 2 & -16 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 8 & -19 \end{bmatrix}$

6. (a) $A + B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1-3 & 2-2 \\ 2+4 & 1+2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1+3 & 2+2 \\ 2-4 & 1-2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -2 & -1 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(2) \\ 3(2) & 3(1) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix} - 2\begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 3+6 & 6+4 \\ 6-8 & 3-4 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ -2 & -1 \end{bmatrix}$

7. $A = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} 7 & 3 \\ 1 & 9 \\ -2 & 15 \end{bmatrix}$ (b) $A - B = \begin{bmatrix} 5 & -5 \\ 3 & -1 \\ -4 & -5 \end{bmatrix}$ (c) $3A = \begin{bmatrix} 18 & -3 \\ 6 & 12 \\ -9 & 15 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 18 & -3 \\ 6 & 12 \\ -9 & 15 \end{bmatrix} - \begin{bmatrix} 2 & 8 \\ -2 & 10 \\ 2 & 20 \end{bmatrix} = \begin{bmatrix} 16 & -11 \\ 8 & 2 \\ -11 & -5 \end{bmatrix}$

8. (a) $A + B = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2+2 & 1-3 & 1+4 \\ -1-3 & -1+1 & 4-2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2-2 & 1-(-3) & 1-4 \\ -1-(-3) & -1-1 & 4-(-2) \end{bmatrix} = \begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 3(2) & 3(1) & 3(1) \\ 3(-1) & 3(-1) & 3(4) \end{bmatrix} = \begin{bmatrix} 6 & 3 & 3 \\ -3 & -3 & 12 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 6 & 3 & 3 \\ -3 & -3 & 12 \end{bmatrix} - 2\begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 3 \\ -3 & -3 & 12 \end{bmatrix} + \begin{bmatrix} -4 & 6 & -8 \\ 6 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 9 & -5 \\ 3 & -5 & 16 \end{bmatrix}$

9. $A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ 1 & 1 & -2 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -1 & 1 & 0 \\ -3 & 4 & 9 & -6 & -7 \end{bmatrix}$

(a) $A + B = \begin{bmatrix} 3 & 3 & -2 & 1 & 1 \\ -2 & 5 & 7 & -6 & -8 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 1 & 1 & 0 & -1 & 1 \\ 4 & -3 & -11 & 6 & 6 \end{bmatrix}$

(c) $3A = \begin{bmatrix} 6 & 6 & -3 & 0 & 3 \\ 3 & 3 & -6 & 0 & -3 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} 6 & 6 & -3 & 0 & 3 \\ 3 & 3 & -6 & 0 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 2 & -2 & 2 & 0 \\ -6 & 8 & 18 & -12 & -14 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -1 & -2 & 3 \\ 9 & -5 & -24 & 12 & 11 \end{bmatrix}$

10. (a) $A + B = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} -1-3 & 4+5 & 0+1 \\ 3+2 & -2-4 & 2-7 \\ 5+10 & 4-9 & -1-1 \\ 0+3 & 8+2 & -6-4 \\ -4+0 & -1+1 & 0-2 \end{bmatrix} = \begin{bmatrix} -4 & 9 & 1 \\ 5 & -6 & -5 \\ 15 & -5 & -2 \\ 3 & 10 & -10 \\ -4 & 0 & -2 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} -1+3 & 4-5 & 0-1 \\ 3-2 & -2+4 & 2+7 \\ 5-10 & 4+9 & -1+1 \\ 0-3 & 8-2 & -6+4 \\ -4-0 & -1-1 & 0+2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 9 \\ -5 & 13 & 0 \\ -3 & 6 & -2 \\ -4 & -2 & 2 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} -1 & 4 & 0 \\ 3 & -2 & 2 \\ 5 & 4 & -1 \\ 0 & 8 & -6 \\ -4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 12 & 0 \\ 9 & -6 & 6 \\ 15 & 12 & -3 \\ 0 & 24 & -18 \\ -12 & -3 & 0 \end{bmatrix}$

(d) $3A - 2B = \begin{bmatrix} -3 & 12 & 0 \\ 9 & -6 & 6 \\ 15 & 12 & -3 \\ 0 & 24 & -18 \\ -12 & -3 & 0 \end{bmatrix} - 2\begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & -7 \\ 10 & -9 & -1 \\ 3 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} -3 & 12 & 0 \\ 9 & -6 & 6 \\ 15 & 12 & -3 \\ 0 & 24 & -18 \\ -12 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 6 & -10 & -2 \\ -4 & 8 & 14 \\ -20 & 18 & 2 \\ -6 & -4 & 8 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & -2 \\ 5 & 2 & 20 \\ -5 & 30 & -1 \\ -6 & 20 & -10 \\ -12 & -5 & 4 \end{bmatrix}$

11. $A = \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix}$

(a) $A + B$ is not possible. A and B do not have the same order.

(b) $A - B$ is not possible. A and B do not have the same order.

(c) $3A = \begin{bmatrix} 18 & 0 & 9 \\ -3 & -12 & 0 \end{bmatrix}$

(d) $3A - 2B$ is not possible. A and B do not have the same order.

12. (a) $A + B$ is not possible. A and B do not have the same order.

(b) $A - B$ is not possible. A and B do not have the same order.

(c) $3A = 3\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$

(d) $3A - 2B$ is not possible. A and B do not have the same order.

13. $\begin{bmatrix} -5 & 0 \\ 3 & -6 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -10 & -8 \\ 14 & 6 \end{bmatrix} = \begin{bmatrix} -5 + 7 + (-10) & 0 + 1 + (-8) \\ 3 + (-2) + 14 & -6 + (-1) + 6 \end{bmatrix} = \begin{bmatrix} -8 & -7 \\ 15 & -1 \end{bmatrix}$

14. $\begin{bmatrix} 6 & 8 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -11 & -7 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 6 + 0 + (-11) & 8 + 5 + (-7) \\ -1 + (-3) + 2 & 0 + (-1) + (-1) \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ -2 & -2 \end{bmatrix}$

15. $4\left(\begin{bmatrix} -4 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -2 \\ 3 & -6 & 0 \end{bmatrix}\right) = 4\begin{bmatrix} -6 & -1 & 3 \\ -3 & 8 & 3 \end{bmatrix} = \begin{bmatrix} -24 & -4 & 12 \\ -12 & 32 & 12 \end{bmatrix}$

16.
$$\begin{aligned} \tfrac{1}{2}([5 \quad -2 \quad 4 \quad 0] + [14 \quad 6 \quad -18 \quad 9]) &= \tfrac{1}{2}[5 + 14 \quad -2 + 6 \quad 4 + (-18) \quad 0 + 9] \\ &= \tfrac{1}{2}[19 \quad 4 \quad -14 \quad 9] \\ &= \left[\tfrac{19}{2} \quad 2 \quad -7 \quad \tfrac{9}{2}\right] \end{aligned}$$

17. $-3\left(\begin{bmatrix} 0 & -3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 8 & 1 \end{bmatrix}\right) - 2\begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix} = -3\begin{bmatrix} -6 & 0 \\ 15 & 3 \end{bmatrix} - \begin{bmatrix} 8 & -8 \\ 14 & -18 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -45 & -9 \end{bmatrix} - \begin{bmatrix} 8 & -8 \\ 14 & -18 \end{bmatrix} = \begin{bmatrix} 10 & 8 \\ -59 & 9 \end{bmatrix}$

18.
$$\begin{aligned} -1\begin{bmatrix} 4 & 11 \\ -2 & -1 \\ 9 & 3 \end{bmatrix} + \tfrac{1}{6}\left(\begin{bmatrix} -5 & -1 \\ 3 & 4 \\ 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -9 & -1 \\ 6 & -1 \end{bmatrix}\right) &= \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \tfrac{1}{6}\begin{bmatrix} -5 + 7 & -1 + 5 \\ 3 + (-9) & 4 + (-1) \\ 0 + 6 & 13 + (-1) \end{bmatrix} \\ &= \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \tfrac{1}{6}\begin{bmatrix} 2 & 4 \\ -6 & 3 \\ 6 & 12 \end{bmatrix} \\ &= \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -1 & \frac{1}{2} \\ 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -4 + \frac{1}{3} & -11 + \frac{2}{3} \\ 2 + (-1) & 1 + \frac{1}{2} \\ -9 + 1 & -3 + 2 \end{bmatrix} = \begin{bmatrix} -\frac{11}{3} & -\frac{31}{3} \\ 1 & \frac{3}{2} \\ -8 & -1 \end{bmatrix} \end{aligned}$$

19. $\frac{3}{7}\begin{bmatrix} 2 & 5 \\ -1 & -4 \end{bmatrix} + 6\begin{bmatrix} -3 & 0 \\ 2 & 2 \end{bmatrix} \approx \begin{bmatrix} -17.143 & 2.143 \\ 11.571 & 10.286 \end{bmatrix}$

20. $55\left(\begin{bmatrix} 14 & -11 \\ -22 & 19 \end{bmatrix} + \begin{bmatrix} -22 & 20 \\ 13 & 6 \end{bmatrix}\right) = \begin{bmatrix} -440 & 495 \\ -495 & 1375 \end{bmatrix}$

21. $-\begin{bmatrix} 3.211 & 6.829 \\ -1.004 & 4.914 \\ 0.055 & -3.889 \end{bmatrix} - \begin{bmatrix} -1.630 & -3.090 \\ 5.256 & 8.335 \\ -9.768 & 4.251 \end{bmatrix} = \begin{bmatrix} -1.581 & -3.739 \\ -4.252 & -13.249 \\ 9.713 & -0.362 \end{bmatrix}$

22. $-12\left(\begin{bmatrix} 6 & 20 \\ 1 & -9 \\ -2 & 5 \end{bmatrix} + \begin{bmatrix} 14 & -15 \\ -8 & -6 \\ 7 & 0 \end{bmatrix} + \begin{bmatrix} -31 & -19 \\ 16 & 10 \\ 24 & -10 \end{bmatrix}\right) = \begin{bmatrix} 132 & 168 \\ -108 & 60 \\ -348 & 60 \end{bmatrix}$

23. $X = 3\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} - 2\begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 3 & 0 \\ 9 & -12 \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ 4 & 0 \\ -8 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -9 \\ -1 & 0 \\ 17 & -10 \end{bmatrix}$

24. $2X = 2A - B$

$$X = A - \tfrac{1}{2}B = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} - \tfrac{1}{2}\begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 0 & \frac{3}{2} \\ 1 & 0 \\ -2 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -2 & -\frac{5}{2} \\ 0 & 0 \\ 5 & -\frac{7}{2} \end{bmatrix}$$

25. $X = -\frac{3}{2}A + \frac{1}{2}B = -\frac{3}{2}\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 3 & \frac{3}{2} \\ -\frac{3}{2} & 0 \\ -\frac{9}{2} & 6 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} \\ 1 & 0 \\ -2 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ -\frac{1}{2} & 0 \\ -\frac{13}{2} & \frac{11}{2} \end{bmatrix}$

26. $2A + 4B = -2X$

$$X = -A - 2B = -1\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} - 2\begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ -4 & 0 \\ 8 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -5 & 0 \\ 5 & 6 \end{bmatrix}$$

27. A is 3×2 and B is 3×3. AB is not possible.

28. A is 2×4, B is 2×2. AB is not possible.

29. A is 3×3, B is $3 \times 2 \implies AB$ is 3×2.

$$\begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} (0)(2) + (-1)(-3) + (0)(1) & (0)(1) + (-1)(4) + (0)(6) \\ (4)(2) + (0)(-3) + (2)(1) & (4)(1) + (0)(4) + (2)(6) \\ (8)(2) + (-1)(-3) + (7)(1) & (8)(1) + (-1)(4) + (7)(6) \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 10 & 16 \\ 26 & 46 \end{bmatrix}$$

30. A is 3×2, B is $2 \times 2 \implies AB$ is 3×2.

$$AB = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 14 \end{bmatrix}$$

31. A is 3×3, B is $3 \times 3 \implies AB$ is 3×3.

$$\begin{bmatrix}1 & 0 & 0\\0 & 4 & 0\\0 & 0 & -2\end{bmatrix}\begin{bmatrix}3 & 0 & 0\\0 & -1 & 0\\0 & 0 & 5\end{bmatrix} = \begin{bmatrix}(1)(3)+(0)(0)+(0)(0) & (1)(0)+(0)(-1)+(0)(0) & (1)(0)+(0)(0)+(0)(5)\\(0)(3)+(4)(0)+(0)(0) & (0)(0)+(4)(-1)+(0)(0) & (0)(0)+(4)(0)+(0)(5)\\(0)(3)+(0)(0)+(-2)(0) & (0)(0)+(0)(-1)+(-2)(0) & (0)(0)+(0)(0)+(-2)(5)\end{bmatrix}$$

$$= \begin{bmatrix}3 & 0 & 0\\0 & -4 & 0\\0 & 0 & -10\end{bmatrix}$$

32. A is 3×3, B is $3 \times 3 \implies AB$ is 3×3.

$$AB = \begin{bmatrix}5 & 0 & 0\\0 & -8 & 0\\0 & 0 & 7\end{bmatrix}\begin{bmatrix}\frac{1}{5} & 0 & 0\\0 & -\frac{1}{8} & 0\\0 & 0 & \frac{1}{2}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & \frac{7}{2}\end{bmatrix}$$

33. A is 3×3, B is $3 \times 3 \implies AB$ is 3×3.

$$\begin{bmatrix}0 & 0 & 5\\0 & 0 & -3\\0 & 0 & 4\end{bmatrix}\begin{bmatrix}6 & -11 & 4\\8 & 16 & 4\\0 & 0 & 0\end{bmatrix} = \begin{bmatrix}(0)(6)+(0)(8)+(5)(0) & (0)(-11)+(0)(16)+(5)(0) & (0)(4)+(0)(4)+(5)(0)\\(0)(6)+(0)(8)+(-3)(0) & (0)(-11)+(0)(16)+(-3)(0) & (0)(4)+(0)(4)+(-3)(0)\\(0)(6)+(0)(8)+(4)(0) & (0)(-11)+(0)(16)+(4)(0) & (0)(4)+(0)(4)+(4)(0)\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

34. A is 2×1, B is $1 \times 4 \implies AB$ is 2×4.

$$\begin{bmatrix}10\\12\end{bmatrix}\begin{bmatrix}6 & -2 & 1 & 6\end{bmatrix} = \begin{bmatrix}60 & -20 & 10 & 60\\72 & -24 & 12 & 72\end{bmatrix}$$

35. $$\begin{bmatrix}5 & 6 & -3\\-2 & 5 & 1\\10 & -5 & 5\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\8 & 1 & 4\\4 & -2 & 9\end{bmatrix} = \begin{bmatrix}41 & 7 & 7\\42 & 5 & 25\\-10 & -25 & 45\end{bmatrix}$$

36. $$\begin{bmatrix}11 & -12 & 4\\14 & 10 & 12\\6 & -2 & 9\end{bmatrix}\begin{bmatrix}12 & 10\\-5 & 12\\15 & 16\end{bmatrix} = \begin{bmatrix}252 & 30\\298 & 452\\217 & 180\end{bmatrix}$$

37. $$\begin{bmatrix}-3 & 8 & -6 & 8\\-12 & 15 & 9 & 6\\5 & -1 & 1 & 5\end{bmatrix}\begin{bmatrix}3 & 1 & 6\\24 & 15 & 14\\16 & 10 & 21\\8 & -4 & 10\end{bmatrix} = \begin{bmatrix}151 & 25 & 48\\516 & 279 & 387\\47 & -20 & 87\end{bmatrix}$$

38. A is 3×3, B is 4×2. AB is not possible.

39. A is 2×4 and B is $2 \times 4 \implies AB$ is not possible.

40. $$\begin{bmatrix}15 & -18\\-4 & 12\\-8 & 22\end{bmatrix}\begin{bmatrix}-7 & 22 & 1\\8 & 16 & 24\end{bmatrix} = \begin{bmatrix}-249 & 42 & -417\\124 & 104 & 284\\232 & 176 & 520\end{bmatrix}$$

41. (a) $$AB = \begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}\begin{bmatrix}2 & -1\\-1 & 8\end{bmatrix} = \begin{bmatrix}(1)(2)+(2)(-1) & (1)(-1)+(2)(8)\\(4)(2)+(2)(-1) & (4)(-1)+(2)(8)\end{bmatrix} = \begin{bmatrix}0 & 15\\6 & 12\end{bmatrix}$$

(b) $$BA = \begin{bmatrix}2 & -1\\-1 & 8\end{bmatrix}\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix} = \begin{bmatrix}(2)(1)+(-1)(4) & (2)(2)+(-1)(2)\\(-1)(1)+(8)(4) & (-1)(2)+(8)(2)\end{bmatrix} = \begin{bmatrix}-2 & 2\\31 & 14\end{bmatrix}$$

(c) $$A^2 = \begin{bmatrix}1 & 2\\4 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\4 & 2\end{bmatrix} = \begin{bmatrix}(1)(1)+(2)(4) & (1)(2)+(2)(2)\\(4)(1)+(2)(4) & (4)(2)+(2)(2)\end{bmatrix} = \begin{bmatrix}9 & 6\\12 & 12\end{bmatrix}$$

42. (a) $AB = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 2(0) + (-1)3 & 2(0) + (-1)(-3) \\ 1(0) + 4(3) & 1(0) + 4(-3) \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 12 & -12 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0(2) + (0)1 & 0(-1) + (0)(4) \\ 3(2) + (-3)(1) & 3(-1) + (-3)4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 3 & -15 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2(2) + (-1)(1) & 2(-1) + (-1)4 \\ 1(2) + 4(1) & 1(-1) + 4(4) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & 15 \end{bmatrix}$

43. (a) $AB = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} (3)(1) + (-1)(3) & (3)(-3) + (-1)(1) \\ (1)(1) + (3)(3) & (1)(-3) + (3)(1) \end{bmatrix} = \begin{bmatrix} 0 & -10 \\ 10 & 0 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} (1)(3) + (-3)(1) & (1)(-1) + (-3)(3) \\ (3)(3) + (1)(1) & (3)(-1) + (1)(3) \end{bmatrix} = \begin{bmatrix} 0 & -10 \\ 10 & 0 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} (3)(3) + (-1)(1) & (3)(-1) + (-1)(3) \\ (1)(3) + (3)(1) & (1)(-1) + (3)(3) \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ 6 & 8 \end{bmatrix}$

44. (a) $AB = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + (-1)(-3) & 1(3) + (-1)(1) \\ 1(1) + 1(-3) & 1(3) + 1(1) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -2 & 4 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + (3)1 & 1(-1) + 3(1) \\ -3(1) + (1)(1) & -3(-1) + 1(1) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -2 & 4 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1(1) + (-1)(1) & 1(-1) + (-1)(1) \\ 1(1) + (1)(1) & 1(-1) + 1(1) \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

45. (a) $AB = \begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 7(1) & 7(1) & 7(2) \\ 8(1) & 8(1) & 8(2) \\ -1(1) & -1(1) & -1(2) \end{bmatrix} = \begin{bmatrix} 7 & 7 & 14 \\ 8 & 8 & 16 \\ -1 & -1 & -2 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix} = [(1)(7) + (1)(8) + (2)(-1)] = [13]$

(c) A^2 is not possible.

46. (a) $AB = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = [3(2) + 2(3) + 1(0)] = [12]$

(b) $BA = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(2) & 2(1) \\ 3(3) & 3(2) & 3(1) \\ 0(3) & 0(2) & 0(1) \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 \\ 9 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

(c) The number of columns of A does not equal the number of rows of A; the multiplication is not possible.

47. $\begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ -4 & -16 \end{bmatrix}$

48. $$3\left(\begin{bmatrix} 6 & 5 & -1 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ -1 & -3 \\ 4 & 1 \end{bmatrix}\right) = -3\left(\begin{bmatrix} 6(0) + 5(-1) + (-1)(4) & 6(3) + 5(-3) + (-1)(1) \\ 1(0) + (-2)(-1) + (0)(4) & 1(3) + (-2)(-3) + (0)(1) \end{bmatrix}\right)$$

$$= -3\begin{bmatrix} -9 & 2 \\ 2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & -6 \\ -6 & -27 \end{bmatrix}$$

49. $$\begin{bmatrix}0&2&-2\\4&1&2\end{bmatrix}\left(\begin{bmatrix}4&0\\0&-1\\-1&2\end{bmatrix}+\begin{bmatrix}-2&3\\-3&5\\0&-3\end{bmatrix}\right)=\begin{bmatrix}0&2&-2\\4&1&2\end{bmatrix}\begin{bmatrix}2&3\\-3&4\\-1&-1\end{bmatrix}=\begin{bmatrix}-4&10\\3&14\end{bmatrix}$$

50. $$\begin{bmatrix}3\\-1\\5\\7\end{bmatrix}([5\quad -6]+[7\quad -1]+[-8\quad 9])=\begin{bmatrix}3\\-1\\5\\7\end{bmatrix}[4\quad 2]$$

$$=\begin{bmatrix}3(4)&3(2)\\(-1)(4)&(-1)(2)\\5(4)&5(2)\\7(4)&7(2)\end{bmatrix}=\begin{bmatrix}12&6\\-4&-2\\20&10\\28&14\end{bmatrix}$$

51. (a) $\begin{bmatrix}-1&1\\-2&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}4\\0\end{bmatrix}$

(b) $$\left[\begin{array}{rrcr}-1&1&\vdots&4\\-2&1&\vdots&0\end{array}\right]$$

$$-R_2+R_1\longrightarrow\left[\begin{array}{rrcr}1&0&\vdots&4\\-2&1&\vdots&0\end{array}\right]$$

$$2R_1+R_2\longrightarrow\left[\begin{array}{rrcr}1&0&\vdots&4\\0&1&\vdots&8\end{array}\right]$$

$$X=\begin{bmatrix}4\\8\end{bmatrix}$$

52. (a) $\begin{bmatrix}2&3\\1&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}5\\10\end{bmatrix}$

(b) $$\begin{array}{r}R_2\\R_1\end{array}\left[\begin{array}{rrcr}1&4&\vdots&10\\2&3&\vdots&5\end{array}\right]$$

$$-2R_1+R_2\longrightarrow\left[\begin{array}{rrcr}1&4&\vdots&10\\0&-5&\vdots&-15\end{array}\right]$$

$$-\tfrac{1}{5}R_2\longrightarrow\left[\begin{array}{rrcr}1&4&\vdots&10\\0&1&\vdots&3\end{array}\right]$$

$$\begin{array}{r}-4R_2+R_1\longrightarrow\\-\frac{1}{5}R_2\longrightarrow\end{array}\left[\begin{array}{rrcr}1&0&\vdots&-2\\0&1&\vdots&3\end{array}\right]$$

$$X=\begin{bmatrix}-2\\3\end{bmatrix}$$

53. (a) $\begin{bmatrix}-2&-3\\6&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-4\\-36\end{bmatrix}$

(b) $$\left[\begin{array}{rrcr}-2&-3&\vdots&-4\\6&1&\vdots&-36\end{array}\right]$$

$$3R_1+R_2\longrightarrow\left[\begin{array}{rrcr}-2&-3&\vdots&-4\\0&-8&\vdots&-48\end{array}\right]$$

$$\begin{array}{r}-\frac{1}{2}R_1\longrightarrow\\-\frac{1}{8}R_2\longrightarrow\end{array}\left[\begin{array}{rrcr}1&\frac{3}{2}&\vdots&2\\0&1&\vdots&6\end{array}\right]$$

$$-\tfrac{3}{2}R_2+R_1\longrightarrow\left[\begin{array}{rrcr}1&0&\vdots&-7\\0&1&\vdots&6\end{array}\right]$$

$$X=\begin{bmatrix}-7\\6\end{bmatrix}$$

54. (a) $\begin{bmatrix}-4&9\\1&-3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-13\\12\end{bmatrix}$

(b) $$\begin{array}{r}R_1\\R_2\end{array}\left[\begin{array}{rrcr}1&-3&\vdots&12\\-4&9&\vdots&-13\end{array}\right]$$

$$4R_1+R_2\longrightarrow\left[\begin{array}{rrcr}1&-3&\vdots&12\\0&-3&\vdots&35\end{array}\right]$$

$$-\tfrac{1}{3}R_2\longrightarrow\left[\begin{array}{rrcr}1&-3&\vdots&12\\0&1&\vdots&-\frac{35}{3}\end{array}\right]$$

$$3R_2+R_1\longrightarrow\left[\begin{array}{rrcr}1&0&\vdots&-23\\0&1&\vdots&-\frac{35}{3}\end{array}\right]$$

$$X=\begin{bmatrix}-23\\-\frac{35}{3}\end{bmatrix}$$

55. (a) $A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 17 \end{bmatrix}$

(b) $$\left[\begin{array}{rrrcr} 1 & -2 & 3 & \vdots & 9 \\ -1 & 3 & -1 & \vdots & -6 \\ 2 & -5 & 5 & \vdots & 17 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \rightarrow \\ -2R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & -2 & 3 & \vdots & 9 \\ 0 & 1 & 2 & \vdots & 3 \\ 0 & -1 & -1 & \vdots & -1 \end{array}\right]$$

$$\begin{array}{r} 2R_2 + R_1 \rightarrow \\ \\ R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 7 & \vdots & 15 \\ 0 & 1 & 2 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 2 \end{array}\right]$$

$$\begin{array}{r} -7R_3 + R_1 \rightarrow \\ -2R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -1 \\ 0 & 0 & 1 & \vdots & 2 \end{array}\right]$$

$X = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

56. (a) $\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -5 \end{bmatrix}$

(b) $$\left[\begin{array}{rrrcr} 1 & 1 & -3 & \vdots & 9 \\ -1 & 2 & 0 & \vdots & 6 \\ 1 & -1 & 1 & \vdots & -5 \end{array}\right]$$

$$\begin{array}{r} \\ R_1 + R_2 \rightarrow \\ -R_1 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 1 & -3 & \vdots & 9 \\ 0 & 3 & -3 & \vdots & 15 \\ 0 & -2 & 4 & \vdots & -14 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{3}R_2 \rightarrow \\ -\frac{1}{2}R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 1 & -3 & \vdots & 9 \\ 0 & 1 & -1 & \vdots & 5 \\ 0 & 1 & -2 & \vdots & 7 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 1 & -3 & \vdots & 9 \\ 0 & 1 & -1 & \vdots & 5 \\ 0 & 0 & -1 & \vdots & 2 \end{array}\right]$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ -R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 0 & -2 & \vdots & 4 \\ 0 & 1 & -1 & \vdots & 5 \\ 0 & 0 & 1 & \vdots & -2 \end{array}\right]$$

$$\begin{array}{r} 2R_3 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 0 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -2 \end{array}\right]$$

$X = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$

57. (a) $\begin{bmatrix} 1 & -5 & 2 \\ -3 & 1 & -1 \\ 0 & -2 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ 8 \\ -16 \end{bmatrix}$

(b) $$\left[\begin{array}{rrrcr} 1 & -5 & 2 & \vdots & -20 \\ -3 & 1 & -1 & \vdots & 8 \\ 0 & -2 & 5 & \vdots & -16 \end{array}\right]$$

$$\begin{array}{r} \\ 3R_1 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & -5 & 2 & \vdots & -20 \\ 0 & -14 & 5 & \vdots & -52 \\ 0 & -2 & 5 & \vdots & -16 \end{array}\right]$$

$$\begin{array}{r} \\ -R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & -5 & 2 & \vdots & -20 \\ 0 & -12 & 0 & \vdots & -36 \\ 0 & -2 & 5 & \vdots & -16 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{1}{12}R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & -5 & 2 & \vdots & -20 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & -2 & 5 & \vdots & -16 \end{array}\right]$$

$$\begin{array}{r} 5R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 2 & \vdots & -5 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 5 & \vdots & -10 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{5}R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 2 & \vdots & -5 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -2 \end{array}\right]$$

$$\begin{array}{r} -2R_3 + R_1 \rightarrow \\ \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -2 \end{array}\right]$$

$X = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$

58. (a) $\begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & 0 \\ 0 & -6 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -11 \\ 40 \end{bmatrix}$

(b) $$\left[\begin{array}{rrrcr} 1 & -1 & 4 & \vdots & 17 \\ 1 & 3 & 0 & \vdots & -11 \\ 0 & -6 & 5 & \vdots & 40 \end{array}\right]$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & -1 & 4 & \vdots & 17 \\ 0 & 4 & -4 & \vdots & -28 \\ 0 & -6 & 5 & \vdots & 40 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{4}R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & -1 & 4 & \vdots & 17 \\ 0 & 1 & -1 & \vdots & -7 \\ 0 & -6 & 5 & \vdots & 40 \end{array}\right]$$

$$\begin{array}{r} \\ \\ 6R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & -1 & 4 & \vdots & 17 \\ 0 & 1 & -1 & \vdots & -7 \\ 0 & 0 & -1 & \vdots & -2 \end{array}\right]$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -R_3 \rightarrow \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 3 & \vdots & 10 \\ 0 & 1 & -1 & \vdots & -7 \\ 0 & 0 & 1 & \vdots & 2 \end{array}\right]$$

$$\begin{array}{r} -3R_3 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcr} 1 & 0 & 0 & \vdots & 4 \\ 0 & 1 & 0 & \vdots & -5 \\ 0 & 0 & 1 & \vdots & 2 \end{array}\right]$$

$X = \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$

59. $1.2\begin{bmatrix} 70 & 50 & 25 \\ 35 & 100 & 70 \end{bmatrix} = \begin{bmatrix} 84 & 60 & 30 \\ 42 & 120 & 84 \end{bmatrix}$

60. $1.10\begin{bmatrix} 100 & 90 & 70 & 30 \\ 40 & 20 & 60 & 60 \end{bmatrix} = \begin{bmatrix} 110 & 99 & 77 & 33 \\ 44 & 22 & 66 & 66 \end{bmatrix}$

61. (a)

$$A = \begin{array}{c} \begin{array}{ccc} \text{Farmer's Market} & \text{Fruit Stand} & \text{Fruit Farm} \end{array} \\ \begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Apples} \\ \text{Peaches} \end{array}$$

Each entry represents the number of bushels of each type of crop that are shipped to each outlet.

(b) $B = [3.50 \quad 6.00]$

Each entry represents the profit per bushel for each type of crop.

(c) $BA = [3.50 \quad 6.00]\begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix}$

$= [\$1037.50 \quad \$1400.00 \quad \$1012.50]$

The entries in the matrix represent the profits for both crops at each of the three outlets.

62. $BA = [\$39.50 \quad \$44.50 \quad \$56.50]\begin{bmatrix} 5{,}000 & 4{,}000 \\ 6{,}000 & 10{,}000 \\ 8{,}000 & 5{,}000 \end{bmatrix} = [\$916{,}500 \quad \$885{,}500]$

The entries represent the costs of the three models of the product at the two warehouses.

63. $ST = \begin{bmatrix} 3 & 2 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 & 3 \\ 4 & 2 & 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 840 & 1100 \\ 1200 & 1350 \\ 1450 & 1650 \\ 2650 & 3000 \\ 3050 & 3200 \end{bmatrix} = \begin{bmatrix} \$15{,}770 & \$18{,}300 \\ \$26{,}500 & \$29{,}250 \\ \$21{,}260 & \$24{,}150 \end{bmatrix}$

The entries represent the wholesale and retail inventory values of the inventories at the three outlets.

64. $P^2 = \begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix}\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.40 & 0.15 & 0.15 \\ 0.28 & 0.53 & 0.17 \\ 0.32 & 0.32 & 0.68 \end{bmatrix}$

The P^2 matrix gives the proportion of the voting population that changed parties or remained loyal to their party from the first election to the third.

65. $P^3 = P^2P = \begin{bmatrix} 0.40 & 0.15 & 0.15 \\ 0.28 & 0.53 & 0.17 \\ 0.32 & 0.32 & 0.68 \end{bmatrix}\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.300 & 0.175 & 0.175 \\ 0.308 & 0.433 & 0.217 \\ 0.392 & 0.392 & 0.608 \end{bmatrix}$

$P^4 = P^3P = \begin{bmatrix} 0.300 & 0.175 & 0.175 \\ 0.308 & 0.433 & 0.217 \\ 0.392 & 0.392 & 0.608 \end{bmatrix}\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.250 & 0.188 & 0.188 \\ 0.315 & 0.377 & 0.248 \\ 0.435 & 0.435 & 0.565 \end{bmatrix}$

$P^5 = P^4P = \begin{bmatrix} 0.250 & 0.188 & 0.188 \\ 0.315 & 0.377 & 0.248 \\ 0.435 & 0.435 & 0.565 \end{bmatrix}\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 0.225 & 0.194 & 0.194 \\ 0.314 & 0.345 & 0.267 \\ 0.461 & 0.461 & 0.539 \end{bmatrix}$

$P^6 = \begin{bmatrix} 0.213 & 0.197 & 0.197 \\ 0.311 & 0.326 & 0.280 \\ 0.477 & 0.477 & 0.523 \end{bmatrix}$

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65. —CONTINUED—

$$P^7 = \begin{bmatrix} 0.206 & 0.198 & 0.198 \\ 0.308 & 0.316 & 0.288 \\ 0.486 & 0.486 & 0.514 \end{bmatrix}$$

$$P^8 = \begin{bmatrix} 0.203 & 0.199 & 0.199 \\ 0.305 & 0.309 & 0.292 \\ 0.492 & 0.492 & 0.508 \end{bmatrix}$$

As P is raised to higher and higher powers, the resulting matrices appear to be approaching the matrix

$$\begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}.$$

66. $ST = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 1.6 & 1.0 & 0.2 \\ 2.5 & 2.0 & 1.4 \end{bmatrix}\begin{bmatrix} 12 & 10 \\ 9 & 8 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} \$18.10 & \$15.40 \\ \$29.80 & \$25.40 \\ \$59.20 & \$50.80 \end{bmatrix}$

This represents the labor cost for each boat size at each plant.

67. (a) $AB = \begin{bmatrix} 40 & 64 & 52 \\ 60 & 82 & 76 \\ 76 & 96 & 84 \end{bmatrix}\begin{bmatrix} 2.65 & 0.65 \\ 2.85 & 0.70 \\ 3.05 & 0.85 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \text{Sales} & \text{Profit} \end{array} \\ \begin{bmatrix} 447 & 115 \\ 624.50 & 161 \\ 731.20 & 188 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Friday} \\ \text{Saturday} \\ \text{Sunday} \end{array}$

The entries in Column 1 represent the total sales of the three kinds of milk for Friday, Saturday, and Sunday. The entries in Column 2 represent each days' total profit.

(b) Total profit for the weekend: $115 + 161 + 188 = \$464$

68. (a) $AB = \begin{bmatrix} 580 & 840 & 320 \\ 560 & 420 & 160 \\ 860 & 1020 & 540 \end{bmatrix}\begin{bmatrix} 1.95 & 0.32 \\ 2.05 & 0.36 \\ 2.15 & 0.40 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \text{Sales (\$)} & \text{Profit} \end{array} \\ \begin{bmatrix} 3541 & 616 \\ 2297 & 394.4 \\ 4929 & 858.4 \end{bmatrix} \end{array} \begin{array}{l} \\ 87 \\ 89 \\ 93 \end{array}$

The first column of AB gives the amount of sales for each octane. The second column gives the profit made by each octane.

(b) The store's profit for the weekend is $\$616 + \$394.40 + \$858.40 = \$1868.80$.

69. (a) $B = \begin{array}{ccc} \text{Bicycled} & \text{Jogged} & \text{Walked} \\ [2 & 0.5 & 3] \end{array}$ 20-minute time periods

(b) $BA = [2 \quad 0.5 \quad 3]\begin{bmatrix} 109 & 136 \\ 127 & 159 \\ 64 & 79 \end{bmatrix} = \begin{array}{cc} \text{120-pound person} & \text{150-pound person} \\ [473.5 & 588.5] \end{array}$ Calories burned

The first entry represents the total calories burned by the 120-pound person and the second entry represents the total calories burned by the 150-pound person.

70. (a)

$A = \begin{array}{c} \begin{array}{cc} \text{Individual costs} & \text{Family costs} \end{array} \\ \begin{bmatrix} 694.32 & 1725.36 \\ 451.8 & 1187.76 \\ 489.48 & 1248.12 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Comprehensive plan} \\ \text{HMO standard plan} \\ \text{HMO plus plan} \end{array}$

$B = \begin{bmatrix} 683.91 & 1699.48 \\ 463.1 & 1217.45 \\ 499.27 & 1273.08 \end{bmatrix} \begin{array}{l} \text{Comprehensive plan} \\ \text{HMO standard plan} \\ \text{HMO plus plan} \end{array}$

—CONTINUED—

70. —CONTINUED—

(b)

$$A - B = \begin{bmatrix} 694.32 & 1725.36 \\ 451.8 & 1187.76 \\ 489.48 & 1248.12 \end{bmatrix} - \begin{bmatrix} 683.91 & 1699.48 \\ 463.1 & 1217.45 \\ 499.27 & 1273.08 \end{bmatrix} = \begin{array}{cc} \text{Change in individual costs} & \text{Change in family cost} \\ \begin{bmatrix} 10.41 & 25.88 \\ -11.3 & -29.69 \\ -9.79 & -24.96 \end{bmatrix} & \begin{array}{l} \text{Comprehensive plan} \\ \text{HMO standard plan} \\ \text{HMO plus plan} \end{array} \end{array}$$

Employees choosing the comprehensive plan have a decrease in cost while those choosing the other two have an increased cost.

(c) Dividing each entry of matrix A by 12 yields

$$\tfrac{1}{12}A = \begin{bmatrix} 57.86 & 143.78 \\ 37.65 & 98.98 \\ 40.79 & 104.01 \end{bmatrix}, \quad \tfrac{1}{12}B = \begin{bmatrix} 56.99 & 141.62 \\ 38.59 & 101.45 \\ 41.61 & 106.09 \end{bmatrix}.$$

(d) If the costs increase by 4% next year, then the new cost matrix would be:

$$A + 0.04A = \begin{bmatrix} 722.09 & 1794.37 \\ 469.87 & 1235.27 \\ 509.06 & 1298.05 \end{bmatrix}$$

$$\tfrac{1}{12}(A + 0.04A) = \begin{array}{cc} \text{Monthly individual cost} \quad \text{Monthly family cost} & \\ \begin{bmatrix} 60.17 & 149.53 \\ 39.16 & 102.94 \\ 42.42 & 108.17 \end{bmatrix} & \begin{array}{l} \text{Comprehensive plan} \\ \text{HMO standard plan} \\ \text{HMO plus plan} \end{array} \end{array}$$

71. True.
The sum of two matrices of different orders is undefined.

72. False. For most matrices, $AB \neq BA$.

For 73–80, A is of order 2×3, B is of order 2×3, C is of order 3×2 and D is of order 2×2.

73. $A + 2C$ is not possible. A and C are not of the same order.

74. $B - 3C$ is not possible. B and C are not of the same order.

75. AB is not possible. The number of columns of A does not equal the number of rows of B.

76. BC is possible. The resulting order is 2×2.

77. $BC - D$ is possible. The resulting order is 2×2.

78. $CB - D$ is not possible. The order of CB is 3×3, but the order of D is 2×2.

79. $D(A - 3B)$ is possible. The resulting order is 2×3.

80. $(BC - D)A$ is possible. The resulting order is 2×3.

81. $AC = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

$BC = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

Thus, $AC = BC$ even though $A \neq B$.

82. $AB = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = O$ and neither A nor B is O.

83. The product of two diagonal matrices of the same order is a diagonal matrix whose entries are the products of the corresponding diagonal entries of A and B.

84. (a) $A^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} (i)(i) + (0)(0) & (i)(0) + (0)(i) \\ (0)(i) + (i)(0) & (0)(0) + (i)(i) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $i^2 = -1$

$A^3 = A^2A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} (-1)(i) + (0)(0) & (-1)(0) + (0)(i) \\ (0)(i) + (-1)(0) & (0)(0) + (-1)(i) \end{bmatrix} = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}$ and $i^3 = -i$

$A^4 = A^3A = \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} (-i)(i) + (0)(0) & (i)(0) + (0)(i) \\ (0)(i) + (-i)(0) & (0)(0) + (-i)(i) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $i^4 = 1$

(b) $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$B^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} (0)(0) + (-i)(i) & (0)(-i) + (-i)(0) \\ (i)(0) + (0)(i) & (i)(-i) + (0)(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$, the identity matrix

85. $3x^2 + 20x - 32 = 0$

$(3x - 4)(x + 8) = 0$

$3x - 4 = 0$ or $x + 8 = 0$

$x = \frac{4}{3}$ or $x = -8$

Solutions: $\frac{4}{3}, -8$

86. $8x^2 - 10x - 3 = 0$

$(2x - 3)(4x + 1) = 0$

$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

$4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$

Solutions: $-\frac{1}{4}, \frac{3}{2}$

87. $4x^3 + 10x^2 - 3x = 0$

$x(4x^2 + 10x - 3) = 0$

$x = 0$ or $4x^2 + 10x - 3 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(4)(-3)}}{2(4)} = \frac{-10 \pm \sqrt{148}}{8}$$

$= \dfrac{-5 \pm \sqrt{37}}{4}$ by the Quadratic Formula

Solutions: $0, \dfrac{-5 \pm \sqrt{37}}{4}$

88. $3x^3 + 22x^2 - 45x = 0$

$x(3x^2 + 22x - 45) = 0$

$x(x + 9)(3x - 5) = 0$

$x = 0$

$x + 9 = 0 \Rightarrow x = -9$

$3x - 5 = 0 \Rightarrow x = \frac{5}{3}$

Solutions: $0, -9, \frac{5}{3}$

89. $3x^3 - 12x^2 + 5x - 20 = 0$

$3x^2(x - 4) + 5(x - 4) = 0$

$(x - 4)(3x^2 + 5) = 0$

$x - 4 = 0$ or $3x^2 + 5 = 0$

$x = 4$ $\qquad x^2 = -\dfrac{5}{3}$

$$x = \pm\sqrt{-\frac{5}{3}} = \pm\frac{\sqrt{15}}{3}i$$

Solutions: $4, \pm\dfrac{\sqrt{15}}{3}i$

90. $2x^3 - 5x^2 - 12x + 30 = 0$

$x^2(2x - 5) - 6(2x - 5) = 0$

$(2x - 5)(x^2 - 6) = 0$

$2x - 5 = 0 \Rightarrow x = \dfrac{5}{2}$

$x^2 - 6 = 0 \Rightarrow x^2 = 6 \Rightarrow x = \pm\sqrt{6}$

$x = \pm\sqrt{6}$

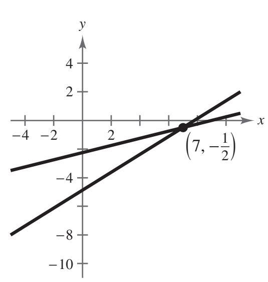
Solutions: $\dfrac{5}{2}, \pm\sqrt{6}$

91. $\begin{cases} -x + 4y = -9 & \text{Eq.1} \\ 5x - 8y = 39 & \text{Eq.2} \end{cases}$

$$\begin{aligned} -5x + 20y &= -45 && \text{(5)Eq.1} \\ 5x - 8y &= 39 && \\ 12y &= -6 && \text{Add equations.} \\ y &= -\tfrac{1}{2} && \end{aligned}$$

$-x + 4\left(-\frac{1}{2}\right) = -9 \Rightarrow x = 7$

Solution: $\left(7, -\frac{1}{2}\right)$

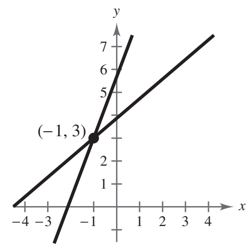


92. $\begin{cases} 8x - 3y = -17 & \text{Equation 1} \\ -6x + 7y = 27 & \text{Equation 2} \end{cases}$

$$\begin{aligned} 48x - 18y &= -102 && (6)\text{Eq.1} \\ -48x + 56y &= 216 && (8)\text{Eq.2} \\ 38y &= 114 && \text{Add equations.} \\ y &= 3 \end{aligned}$$

$8x - 3(3) = -17 \implies x = -1$

Solution: $(-1, 3)$

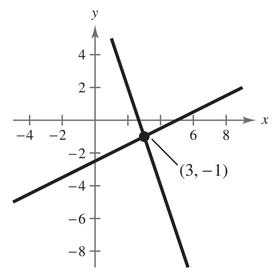


93. $\begin{cases} -x + 2y = -5 & \text{Equation 1} \\ -3x - y = -8 & \text{Equation 2} \end{cases}$

$$\begin{aligned} -x + 2y &= -5 \\ \underline{-6x - 2y} &= \underline{-16} && (2)\text{Eq.2} \\ -7x \quad &= -21 && \text{Add equations.} \\ x &= 3 \end{aligned}$$

$-3 + 2y = -5 \implies y = -1$

Solution: $(3, -1)$

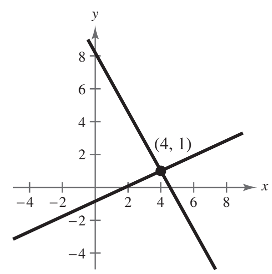


94. $\begin{cases} 6x - 13y = 11 & \text{Equation 1} \\ 9x + 5y = 41 & \text{Equation 2} \end{cases}$

$$\begin{aligned} 18x - 39y &= 33 && (3)\text{Eq.1} \\ \underline{-18x - 10y} &= \underline{-82} && (-2)\text{Eq.2} \\ -49y &= -49 && \text{Add equations.} \\ y &= 1 \end{aligned}$$

$6x - 13(1) = 11 \implies x = 4$

Solution: $(4, 1)$



Section 8.3 The Inverse of a Square Matrix

- You should know that the inverse of an $n \times n$ matrix A is the $n \times n$ matrix A^{-1}, if is exists, such that $AA^{-1} = A^{-1}A = I$, where I is the $n \times n$ identity matrix.
- You should be able to find the inverse, if it exists, of a square matrix.
  - (a) Write the $n \times 2n$ matrix that consists of the given matrix A on the left and the $n \times n$ identity matrix I on the right to obtain $[A \ \vdots \ I]$. Note that we separate the matrices A and I by a dotted line. We call this process **adjoining** the matrices A and I.
  - (b) If possible, row reduce A to I using elementary row operations on the *entire* matrix $[A \ \vdots \ I]$. The result will be the matrix $[I \ \vdots \ A^{-1}]$. If this is not possible, then A is not invertible.
  - (c) Check your work by multiplying to see that $AA^{-1} = I = A^{-1}A$.
- The inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ if $ad - cb \neq 0$.
- You should be able to use inverse matrices to solve systems of linear equations if the coefficient matrix is square and invertible.

Vocabulary Check

1. square **2.** inverse **3.** nonsingular; singular **4.** $A^{-1}B$

1. $AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 6-5 & 3-3 \\ -10+10 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $AB = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2-1 & 1-1 \\ -2+2 & -1+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & -2+2 \\ 1-1 & -1+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -2+3 & 1-1 \\ -6+6 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2+3 & -4+4 \\ \frac{3}{2}-\frac{3}{2} & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. $AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{3}{5}+\frac{2}{5} & \frac{1}{5}-\frac{1}{5} \\ \frac{6}{5}-\frac{6}{5} & \frac{2}{5}+\frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{5}+\frac{2}{5} & -\frac{3}{5}+\frac{3}{5} \\ -\frac{2}{5}+\frac{2}{5} & \frac{2}{5}+\frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. $AB = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} = \begin{bmatrix} 2-34+33 & 2-68+66 & 4+51-55 \\ -1+22-21 & -1+44-42 & -2-33+35 \\ 6-6 & 12-12 & -9+10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 2-1 & -17+11+6 & 11-7-4 \\ 4-4 & -34+44-9 & 22-28+6 \\ 6-6 & -51+66-15 & 33-42+10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. $AB = \begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{4} & -1 & -\frac{11}{4} \\ -\frac{1}{4} & 1 & \frac{7}{4} \end{bmatrix} = \begin{bmatrix} 2+\frac{1}{4}-\frac{5}{4} & -4-1+5 & -6-\frac{11}{4}+\frac{35}{4} \\ \frac{1}{2}+\frac{1}{2}-1 & -1-2+4 & -\frac{3}{2}-\frac{11}{2}+7 \\ -\frac{1}{4}+\frac{1}{4} & 1-1 & \frac{11}{4}-\frac{7}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} -\frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{4} & -1 & -\frac{11}{4} \\ -\frac{1}{4} & 1 & \frac{7}{4} \end{bmatrix}\begin{bmatrix} -4 & 1 & 5 \\ -1 & 2 & 4 \\ 0 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2-1 & -\frac{1}{2}+2-\frac{3}{2} & -\frac{5}{2}+4-\frac{3}{2} \\ -1+1 & \frac{1}{4}-2+\frac{11}{4} & \frac{5}{4}-4+\frac{11}{4} \\ 1-1 & -\frac{1}{4}+2-\frac{7}{4} & -\frac{5}{4}+4-\frac{7}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. $AB = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 4 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & -1 & -1 \\ -4 & 9 & -5 & -6 \\ 0 & 1 & -1 & -1 \\ 3 & -5 & 3 & 3 \end{bmatrix}$

$= \begin{bmatrix} -2+3 & 4+1-5 & -2-1+3 & -2-1+3 \\ 0 & 6-5 & 0 & 0 \\ 1-4+3 & -2+9-2-5 & 1-5+2+3 & 1-6+2+3 \\ 0 & 8-9+1 & -4+5-1 & -4+6-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

—CONTINUED—

7. —CONTINUED—

$$BA = \begin{bmatrix} -1 & 2 & -1 & -1 \\ -4 & 9 & -5 & -6 \\ 0 & 1 & -1 & -1 \\ 3 & -5 & 3 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 4 & -1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6+1-4 & 0 & -1+2-1 & -1+2-1 \\ -8+27+5-24 & -5+6 & -4+10-6 & -4+9-5 \\ 3+1-4 & 0 & 2-1 & 0 \\ 6-15-3+12 & 0 & 3-6+3 & 3-5+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8. $$AB = \begin{bmatrix} -2 & 0 & 1 & 0 \\ 1 & -1 & -3 & 0 \\ -2 & -1 & 0 & -2 \\ 0 & 1 & 3 & -1 \end{bmatrix}\begin{bmatrix} -3 & -3 & 1 & -2 \\ 12 & 14 & -5 & 10 \\ -5 & -6 & 2 & -4 \\ -3 & -4 & 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 6-5 & 6-6 & -2+2 & 4-4 \\ -3-12+15 & -3-14+18 & 1+5-6 & -2-10+12 \\ 6-12+6 & 6-14+8 & -2+5-2 & 4-10+6 \\ 12-15+3 & 14-18+4 & -5+6-1 & 10-12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & -3 & 1 & -2 \\ 12 & 14 & -5 & 10 \\ -5 & -6 & 2 & -4 \\ -3 & -4 & 1 & -3 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 & 0 \\ 1 & -1 & -3 & 0 \\ -2 & -1 & 0 & -2 \\ 0 & 1 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-3-2 & 3-1-2 & -3+9-6 & -2+2 \\ -24+14+10 & -14+5+10 & 12-42+30 & 10-10 \\ 10-6-4 & 6-2-4 & -5+18-12 & -4+4 \\ 6-4-2 & 4-1-3 & -3+12-9 & -2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. $$AB = \frac{1}{3}\begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 8-8+3 & 10-16+6 & -6+6 \\ -4+4 & -5+8 & 3-3 \\ -4+4 & -8+8 & 3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 8-5 & -8+5+3 & -12+12 \\ 8-8 & -8+8+3 & -12+12 \\ -2+2 & 2-2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10. $AB = \frac{1}{3}\begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3-3+3 & -1-1+2 & -1+2-1 & 3-3 \\ -3+3 & 1+1+1 & 1-2+1 & -3+3 \\ 3-3 & -1-1+2 & -1+2+2 & 3-3 \\ 3-3 & 1+1-2 & -2+1+1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$BA = \frac{1}{3}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3+1-1 & -3-1+1+3 & 1+2-3 & 3-3 \\ 3-1-2 & -3+1+2+3 & -1+4-3 & 3-3 \\ 1-1 & -1+1 & 1+2 & 0 \\ 3-2-1 & -3+2+1 & -2+2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

11. $[A \ \vdots \ I] = \left[\begin{array}{cc:cc} 2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{array}\right]$

$\begin{array}{l} \frac{1}{2}R_1 \to \\ \frac{1}{3}R_2 \to \end{array} \left[\begin{array}{cc:cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{3} \end{array}\right] = [I \ \vdots \ A^{-1}]$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$

12. $[A \ \vdots \ I] = \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$

$-3R_1 + R_2 \to \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right]$

$-2R_2 + R_1 \to \left[\begin{array}{cc:cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right] = [I \ \vdots \ A^{-1}]$

$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

13. $[A \ \vdots \ I] = \left[\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 2 & -3 & 0 & 1 \end{array}\right]$

$-2R_1 + R_2 \to \left[\begin{array}{cc:cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$

$2R_2 + R_1 \to \left[\begin{array}{cc:cc} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{array}\right] = [I \ \vdots A^{-1}]$

$A^{-1} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$

14. $[A \ \vdots \ I] = \left[\begin{array}{cc:cc} -7 & 33 & 1 & 0 \\ 4 & -19 & 0 & 1 \end{array}\right]$

$2R_2 + R_1 \to \left[\begin{array}{cc:cc} 1 & -5 & 1 & 2 \\ 4 & -19 & 0 & 1 \end{array}\right]$

$-4R_1 + R_2 \to \left[\begin{array}{cc:cc} 1 & -5 & 1 & 2 \\ 0 & 1 & -4 & -7 \end{array}\right]$

$5R_2 + R_1 \to \left[\begin{array}{cc:cc} 1 & 0 & -19 & -33 \\ 0 & 1 & -4 & -7 \end{array}\right] = [I \ \vdots \ A^{-1}]$

$A^{-1} = \begin{bmatrix} -19 & -33 \\ -4 & -7 \end{bmatrix}$

15. $[A \;\vdots\; I] = \left[\begin{array}{rr|rr} -1 & 1 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{array}\right]$

$-R_2 + R_1 \rightarrow \left[\begin{array}{rr|rr} 1 & 0 & 1 & -1 \\ -2 & 1 & 0 & 1 \end{array}\right]$

$2R_1 + R_2 \rightarrow \left[\begin{array}{rr|rr} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & -1 \end{array}\right] = [I \;\vdots\; A^{-1}]$

$A^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

16. $[A \;\vdots\; I] = \left[\begin{array}{rr|rr} 11 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array}\right]$

$10R_2 + R_1 \rightarrow \left[\begin{array}{rr|rr} 1 & 1 & 1 & 10 \\ -1 & 0 & 0 & 1 \end{array}\right]$

$R_1 + R_2 \rightarrow \left[\begin{array}{rr|rr} 1 & 1 & 1 & 10 \\ 0 & 1 & 1 & 11 \end{array}\right]$

$-R_2 + R_1 \rightarrow \left[\begin{array}{rr|rr} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 11 \end{array}\right] = [I \;\vdots\; A^{-1}]$

$A^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 11 \end{bmatrix}$

17. $[A \;\vdots\; I] = \left[\begin{array}{rr|rr} 2 & 4 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right]$

$-2R_1 + R_2 \rightarrow \left[\begin{array}{rr|rr} 2 & 4 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$

The two zeros in the second row imply that the inverse does not exist.

18. $[A \;\vdots\; I] = \left[\begin{array}{rr|rr} 2 & 3 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right]$

$\begin{array}{r} R_2 \\ R_1 \end{array} \left[\begin{array}{rr|rr} 1 & 4 & 0 & 1 \\ 2 & 3 & 1 & 0 \end{array}\right]$

$-2R_1 + R_2 \rightarrow \left[\begin{array}{rr|rr} 1 & 4 & 0 & 1 \\ 0 & -5 & 1 & -2 \end{array}\right]$

$-\frac{1}{5}R_2 \rightarrow \left[\begin{array}{rr|rr} 1 & 4 & 0 & 1 \\ 0 & 1 & -\frac{1}{5} & \frac{2}{5} \end{array}\right]$

$-4R_2 + R_1 \rightarrow \left[\begin{array}{rr|rr} 1 & 0 & \frac{4}{5} & -\frac{3}{5} \\ 0 & 1 & -\frac{1}{5} & \frac{2}{5} \end{array}\right] = [I \;\vdots\; A^{-1}]$

$A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

19. $A = \begin{bmatrix} 2 & 7 & 1 \\ -3 & -9 & 2 \end{bmatrix}$ A has no inverse because it is not square.

20. $A = \begin{bmatrix} -2 & 5 \\ 6 & -15 \\ 0 & 1 \end{bmatrix}$ A has no inverse because it is not square.

21. $[A \;\vdots\; I] = \left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right]$

$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right]$

$\frac{1}{2}R_2 \rightarrow \left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 3 & 2 & -3 & 0 & 1 \end{array}\right]$

$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ -3R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & \frac{1}{2} & \frac{5}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right]$

$\begin{array}{r} -R_3 + R_1 \rightarrow \\ -R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & \frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 1 \end{array}\right]$

$\begin{array}{r} \\ \\ 2R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right] = [I \;\vdots\; A^{-1}]$

$A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}$

22. $[A \ \vdots \ I] = \left[\begin{array}{rrr|rrr} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} -2R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & -4 & 7 & -2 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$$

$$\begin{array}{r} 4R_3 + R_1 \rightarrow \\ -3R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

23. $[A \ \vdots \ I] = \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & -3 & 1 & 0 \\ 0 & 5 & 5 & -2 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{5}{4}R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & -3 & 1 & 0 \\ 0 & 0 & 5 & \frac{7}{4} & -\frac{5}{4} & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{4}R_2 \rightarrow \\ \frac{1}{5}R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{7}{20} & -\frac{1}{4} & \frac{1}{5} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & \frac{1}{4} & 0 \\ \frac{7}{20} & -\frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

24. $[A \ \vdots \ I] = \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 1 & 0 \\ 0 & 5 & 5 & -2 & 0 & 1 \end{array}\right]$

Since the first three entries of row 2 are all zeros, the inverse of A does not exist.

25. $[A \ \vdots \ I] = \left[\begin{array}{rrrr|rrrr} -8 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{r} -\frac{1}{8}R_1 \rightarrow \\ \\ \frac{1}{4}R_3 \rightarrow \\ -\frac{1}{5}R_4 \rightarrow \end{array} \left[\begin{array}{rrrr|rrrr} 1 & 0 & 0 & 0 & -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{5} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{8} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -\frac{1}{5} \end{bmatrix}$$

26. $[A \,\vdots\, I] = \left[\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \longrightarrow \\ \\ \frac{1}{5}R_4 \longrightarrow \end{array} \left[\begin{array}{cccc|cccc} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} -3R_2 + R_1 \longrightarrow \\ R_3 + R_2 \longrightarrow \\ -R_4 + R_3 \longrightarrow \\ \\ \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & -8 & -9 & 1 & -\frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 & 4 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} -4R_3 + R_1 \longrightarrow \\ -4R_4 + R_2 \longrightarrow \\ -\frac{1}{2}R_3 \longrightarrow \\ \\ \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -9 & 1 & -\frac{3}{2} & -4 & \frac{4}{5} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right]$$

$$\begin{array}{r} 9R_4 + R_1 \longrightarrow \\ \\ \\ \\ \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -\frac{3}{2} & -4 & \frac{13}{5} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{4}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{10} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{5} \end{array}\right] = [I \,\vdots\, A^{-1}]$$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 10 & -15 & -40 & 26 \\ 0 & 5 & 10 & -8 \\ 0 & 0 & -5 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

27. $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$

28. $A = \begin{bmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -10 & -4 & 27 \\ 2 & 1 & -5 \\ -13 & -5 & 35 \end{bmatrix}$

29. $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & 3 & 2 \\ 9 & -7 & -6 \\ -2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} -1.5 & 1.5 & 1 \\ 4.5 & -3.5 & -3 \\ -1 & 1 & 1 \end{bmatrix}$

30. $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 7 & -8.5 & 1 \\ -8 & 10 & -1 \end{bmatrix}$

31. $A = \begin{bmatrix} -\frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 1 & 0 & -\frac{3}{2} \\ 0 & -1 & \frac{1}{2} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -12 & -5 & -9 \\ -4 & -2 & -4 \\ -8 & -4 & -6 \end{bmatrix}$

32. $\begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} \\ 0 & \frac{2}{3} & 2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$

A^{-1} does not exist.

33. $A = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$

$A^{-1} = \frac{5}{11}\begin{bmatrix} 0 & -4 & 2 \\ -22 & 11 & 11 \\ 22 & -6 & -8 \end{bmatrix} = \begin{bmatrix} 0 & -1.\overline{81} & 0.\overline{90} \\ -10 & 5 & 5 \\ 10 & -2.\overline{72} & -3.\overline{63} \end{bmatrix}$

34. $A = \begin{bmatrix} 0.6 & 0 & -0.3 \\ 0.7 & -1 & 0.2 \\ 1 & 0 & -0.9 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3.75 & 0 & -1.25 \\ 3.458\overline{3} & -1 & -1.375 \\ 4.1\overline{6} & 0 & -2.5 \end{bmatrix}$

35. $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \\ 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix}$

A^{-1} does not exist.

36. $A = \begin{bmatrix} 4 & 8 & -7 & 14 \\ 2 & 5 & -4 & 6 \\ 0 & 2 & 1 & -7 \\ 3 & 6 & -5 & 10 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 27 & -10 & 4 & -29 \\ -16 & 5 & -2 & 18 \\ -17 & 4 & -2 & 20 \\ -7 & 2 & -1 & 8 \end{bmatrix}$

37. $A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

38. $A = \begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix}$

39. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A = \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix}$

$ad - bc = (5)(3) - (-2)(2) = 19$

$A^{-1} = \dfrac{1}{19}\begin{bmatrix} 3 & 2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ -\frac{2}{19} & \frac{5}{19} \end{bmatrix}$

40. $A = \begin{bmatrix} 7 & 12 \\ -8 & -5 \end{bmatrix}$

$ad - bc = 7(-5) - 12(-8) = -35 + 96 = 61$

$A^{-1} = \dfrac{1}{61}\begin{bmatrix} -5 & -12 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} -\frac{5}{61} & -\frac{12}{61} \\ \frac{8}{61} & \frac{7}{61} \end{bmatrix}$

41. $A = \begin{bmatrix} -4 & -6 \\ 2 & 3 \end{bmatrix}$

$ad - bc = (-4)(3) - (-2)(-6) = 0$

Since $ad - bc = 0$, A^{-1} does not exist.

42. $A = \begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}$

$ad - bc = (-12)(-2) - 3(5) = 24 - 15 = 9$

$A^{-1} = \dfrac{1}{9}\begin{bmatrix} -2 & -3 \\ -5 & -12 \end{bmatrix} = \begin{bmatrix} -\frac{2}{9} & -\frac{1}{3} \\ -\frac{5}{9} & -\frac{4}{3} \end{bmatrix}$

43. $A = \begin{bmatrix} \frac{7}{2} & -\frac{3}{4} \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}$

$ad - bc = \left(\dfrac{7}{2}\right)\left(\dfrac{4}{5}\right) - \left(-\dfrac{3}{4}\right)\left(\dfrac{1}{5}\right) = \dfrac{28}{10} + \dfrac{3}{20} = \dfrac{59}{20}$

$A^{-1} = \dfrac{1}{59/20}\begin{bmatrix} \frac{4}{5} & \frac{3}{4} \\ -\frac{1}{5} & \frac{7}{2} \end{bmatrix} = \dfrac{20}{59}\begin{bmatrix} \frac{4}{5} & \frac{3}{4} \\ -\frac{1}{5} & \frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{16}{59} & \frac{15}{59} \\ -\frac{4}{59} & \frac{70}{59} \end{bmatrix}$

44. $A = \begin{bmatrix} -\frac{1}{4} & \frac{9}{4} \\ \frac{5}{3} & \frac{8}{9} \end{bmatrix}$

$ad - bc = \left(-\dfrac{1}{4}\right)\left(\dfrac{8}{9}\right) - \left(\dfrac{9}{4}\right)\left(\dfrac{5}{3}\right) = -\dfrac{143}{36}$

$A^{-1} = -\dfrac{36}{143}\begin{bmatrix} \frac{8}{9} & -\frac{9}{4} \\ -\frac{5}{3} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} -\frac{32}{143} & \frac{81}{143} \\ \frac{60}{143} & \frac{9}{143} \end{bmatrix}$

45. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

Solution: $(5, 0)$

46. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

Solution: $(6, 3)$

47. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$

Solution: $(-8, -6)$

48. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$

Solution: $(-7, -4)$

49. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ -11 \end{bmatrix}$

Solution: $(3, 8, -11)$

50. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -9 \end{bmatrix}$

Solution: $(1, 7, -9)$

51. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Solution: $(2, 1, 0, 0)$

52. $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -24 & 7 & 1 & -2 \\ -10 & 3 & 0 & -1 \\ -29 & 7 & 3 & -2 \\ 12 & -3 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} -32 \\ -13 \\ -37 \\ 15 \end{bmatrix}$

Solution: $(-32, -13, -37, 15)$

53. $A = \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$

$A^{-1} = \dfrac{1}{9 - 20}\begin{bmatrix} 3 & -4 \\ -5 & 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{11}\begin{bmatrix} 3 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = -\dfrac{1}{11}\begin{bmatrix} -22 \\ 22 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

Solution: $(2, -2)$

54. $A = \begin{bmatrix} 18 & 12 \\ 30 & 24 \end{bmatrix}$

$A^{-1} = \dfrac{1}{432 - 360}\begin{bmatrix} 24 & -12 \\ -30 & 18 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{72}\begin{bmatrix} 24 & -12 \\ -30 & 18 \end{bmatrix}\begin{bmatrix} 13 \\ 23 \end{bmatrix} = \dfrac{1}{72}\begin{bmatrix} 36 \\ 24 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$

Solution: $\left(\frac{1}{2}, \frac{1}{3}\right)$

55. $A = \begin{bmatrix} -0.4 & 0.8 \\ 2 & -4 \end{bmatrix}$

$A^{-1} = \dfrac{1}{1.6 - 1.6}\begin{bmatrix} -4 & -0.8 \\ -2 & -0.4 \end{bmatrix}$

A^{-1} does not exist.

This implies that there is no unique solution; that is, either the system is inconsistent *or* there are infinitely many solutions.

Find the reduced row-echelon form of the matrix corresponding to the system.

$$\begin{bmatrix} -0.4 & 0.8 & \vdots & 1.6 \\ 2 & -4 & \vdots & 5 \end{bmatrix}$$

$$-2.5R_1 \rightarrow \begin{bmatrix} 1 & -2 & \vdots & -4 \\ 2 & -4 & \vdots & 5 \end{bmatrix}$$

$$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -2 & \vdots & -4 \\ 0 & 0 & \vdots & 13 \end{bmatrix}$$

The given system is inconsistent and there is no solution.

56. $A = \begin{bmatrix} 0.2 & -0.6 \\ -1 & 1.4 \end{bmatrix}$

$A^{-1} = \dfrac{1}{0.28 - 0.6}\begin{bmatrix} 1.4 & 0.6 \\ 1 & 0.2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\dfrac{1}{0.32}\begin{bmatrix} 1.4 & 0.6 \\ 1 & 0.2 \end{bmatrix}\begin{bmatrix} 2.4 \\ -8.8 \end{bmatrix}$

$= -\dfrac{1}{0.32}\begin{bmatrix} -1.92 \\ 0.64 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$

Solution: $(6, -2)$

57. $A = \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$

$$A^{-1} = \frac{1}{-\frac{3}{16} - \frac{9}{16}} \begin{bmatrix} \frac{3}{4} & -\frac{3}{8} \\ -\frac{3}{2} & -\frac{1}{4} \end{bmatrix} = -\frac{4}{3} \begin{bmatrix} \frac{3}{4} & -\frac{3}{8} \\ -\frac{3}{2} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{2} \\ 2 & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{2} \\ 2 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -2 \\ -12 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \end{bmatrix}$$

Solution: $(-4, -8)$

58. $A = \begin{bmatrix} \frac{5}{6} & -1 \\ \frac{4}{3} & -\frac{7}{2} \end{bmatrix}$

$$A^{-1} = \frac{1}{-\frac{35}{12} + \frac{4}{3}} \begin{bmatrix} -\frac{7}{2} & 1 \\ -\frac{4}{3} & \frac{5}{6} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{12}{19} \begin{bmatrix} -\frac{7}{2} & 1 \\ -\frac{4}{3} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} -20 \\ -51 \end{bmatrix} = -\frac{12}{19} \begin{bmatrix} 19 \\ -\frac{95}{6} \end{bmatrix} = \begin{bmatrix} -12 \\ 10 \end{bmatrix}$$

Solution: $(-12, 10)$

59. $A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -2 & 6 \end{bmatrix}$

Find A^{-1}.

$$[A \;\vdots\; I] = \left[\begin{array}{ccc|ccc} 4 & -1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 5 & -2 & 6 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} R_1 \\ \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 5 & -2 & 6 & 0 & 0 & 1 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 4 & -1 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} -R_3 + R_1 \rightarrow \\ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 5 & -1 & 0 & 1 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 4 & -1 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -4R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 5 & -1 & 0 & 1 \\ 0 & 4 & -7 & 2 & 1 & -2 \\ 0 & 3 & -19 & 5 & 0 & -4 \end{array}\right]$$

$$\begin{array}{r} \\ -R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 5 & -1 & 0 & 1 \\ 0 & 1 & 12 & -3 & 1 & 2 \\ 0 & 3 & -19 & 5 & 0 & -4 \end{array}\right]$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -3R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 17 & -4 & 1 & 3 \\ 0 & 1 & 12 & -3 & 1 & 2 \\ 0 & 0 & -55 & 14 & -3 & -10 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{1}{55}R_3 \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 17 & -4 & 1 & 3 \\ 0 & 1 & 12 & -3 & 1 & 2 \\ 0 & 0 & 1 & -\frac{14}{55} & \frac{3}{55} & \frac{2}{11} \end{array}\right]$$

$$\begin{array}{r} -17R_3 + R_1 \rightarrow \\ -12R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{18}{55} & \frac{4}{55} & -\frac{1}{11} \\ 0 & 1 & 0 & \frac{3}{55} & \frac{19}{55} & -\frac{2}{11} \\ 0 & 0 & 1 & -\frac{14}{55} & \frac{3}{55} & \frac{2}{11} \end{array}\right] = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \frac{1}{55} \begin{bmatrix} 18 & 4 & -5 \\ 3 & 19 & -10 \\ -14 & 3 & 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{55} \begin{bmatrix} 18 & 4 & -5 \\ 3 & 19 & -10 \\ -14 & 3 & 10 \end{bmatrix} \begin{bmatrix} -5 \\ 10 \\ 1 \end{bmatrix} = \frac{1}{55} \begin{bmatrix} -55 \\ 165 \\ 110 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

Solution: $(-1, 3, 2)$

60. $A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{82}\begin{bmatrix} -21 & 19 & 16 \\ -44 & 32 & 14 \\ 26 & -4 & -12 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{82}\begin{bmatrix} -21 & 19 & 16 \\ -44 & 32 & 14 \\ 26 & -4 & -12 \end{bmatrix}\begin{bmatrix} -2 \\ 16 \\ 4 \end{bmatrix} = \dfrac{1}{82}\begin{bmatrix} 410 \\ 656 \\ -164 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ -2 \end{bmatrix}$

Solution: $(5, 8, -2)$

61. $A = \begin{bmatrix} 5 & -3 & 2 \\ 2 & 2 & -3 \\ 1 & -7 & 8 \end{bmatrix}$

A^{-1} does not exist. This implies that there is no unique solution; that is, either the system is inconsistent *or* the system has infinitely many solutions. Use a graphing utility to find the reduced row-echelon form of the matrix corresponding to the system.

$\begin{bmatrix} 5 & -3 & 2 & \vdots & 2 \\ 2 & 2 & -3 & \vdots & 3 \\ 1 & -7 & 8 & \vdots & -4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -\frac{5}{16} & \vdots & \frac{13}{16} \\ 0 & 1 & -\frac{19}{16} & \vdots & \frac{11}{16} \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$

$\begin{cases} x - \frac{5}{16}z = \frac{13}{16} \\ y - \frac{19}{16}z = \frac{11}{16} \end{cases}$

Let $z = a$. Then $x = \frac{5}{16}a + \frac{13}{16}$ and $y = \frac{19}{16}a + \frac{11}{16}$.

Solution: $\left(\frac{5}{16}a + \frac{13}{16}, \frac{19}{16}a + \frac{11}{16}, a\right)$ where a is a real number

62. $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 5 & 9 \\ 5 & 9 & 17 \end{bmatrix}$

A^{-1} does not exist. This implies that there is no unique solution; that is, either the system is inconsistent *or* the system has infinitely many solutions. Use a graphing utility to find the reduced row-echelon form of the matrix corresponding to the system.

$\begin{bmatrix} 2 & 3 & 5 & \vdots & 4 \\ 3 & 5 & 9 & \vdots & 7 \\ 5 & 9 & 17 & \vdots & 13 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 & \vdots & -1 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$

$\begin{cases} x - 2z = -1 \\ y + 3z = 2 \end{cases}$

Let $z = a$. Then $x = 2a - 1$ and $y = -3a + 2$.

Solution: $(2a - 1, -3a + 2, a)$ where a is a real number

63. $A = \begin{bmatrix} 3 & -2 & 1 \\ -4 & 1 & -3 \\ 1 & -5 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0.56 & 0.12 & -0.2 \\ -0.04 & -0.08 & -0.2 \\ -0.76 & -0.52 & 0.2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.56 & 0.12 & -0.2 \\ -0.04 & -0.08 & -0.2 \\ -0.76 & -0.52 & 0.2 \end{bmatrix}\begin{bmatrix} -29 \\ 37 \\ -24 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ -2 \end{bmatrix}$

Solution: $(-7, 3, -2)$

64. $A = \begin{bmatrix} -8 & 7 & -10 \\ 12 & 3 & -5 \\ 15 & -9 & 2 \end{bmatrix}$

$A^{-1} \approx \begin{bmatrix} -0.034 & 0.066 & -0.004 \\ -0.086 & 0.117 & -0.139 \\ -0.133 & 0.029 & -0.094 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \approx \begin{bmatrix} -0.034 & 0.066 & -0.004 \\ -0.086 & 0.117 & -0.139 \\ -0.133 & 0.029 & -0.094 \end{bmatrix}\begin{bmatrix} -151 \\ 86 \\ 187 \end{bmatrix} \approx \begin{bmatrix} 10 \\ -3 \\ 5 \end{bmatrix}$

Solution: $(10, -3, 5)$

65. $A = \begin{bmatrix} 7 & -3 & 0 & 2 \\ -2 & 1 & 0 & -1 \\ 4 & 0 & 1 & -2 \\ -1 & 1 & 0 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & -5 & 0 & 3 \\ -2 & -4 & 1 & -2 \\ -1 & -4 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & -5 & 0 & 3 \\ -2 & -4 & 1 & -2 \\ -1 & -4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 41 \\ -13 \\ 12 \\ -8 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -2 \\ 3 \end{bmatrix}$

Solution: $(5, 0, -2, 3)$

66. $A = \begin{bmatrix} 2 & 5 & 0 & 1 \\ 1 & 4 & 2 & -2 \\ 2 & -2 & 5 & 1 \\ 1 & 0 & 0 & -3 \end{bmatrix}$

$$A^{-1} \approx \begin{bmatrix} 0.338 & -0.352 & 0.141 & 0.394 \\ 0.042 & 0.164 & -0.066 & -0.117 \\ -0.141 & 0.230 & 0.108 & -0.164 \\ 0.113 & -0.117 & 0.047 & -0.202 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \approx \begin{bmatrix} 0.338 & -0.352 & 0.141 & 0.394 \\ 0.042 & 0.164 & -0.066 & -0.117 \\ -0.141 & 0.230 & 0.108 & -0.164 \\ 0.113 & -0.117 & 0.047 & -0.202 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ 3 \\ -1 \end{bmatrix} \approx \begin{bmatrix} 6.21 \\ -0.77 \\ -2.67 \\ 2.40 \end{bmatrix}$$

Solution: $(6.21, -0.77, -2.67, 2.40)$

67. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0.065 & 0.07 & 0.09 \\ 0 & 2 & -1 \end{bmatrix}$

$$[A \ \vdots \ I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0.065 & 0.07 & 0.09 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$200R_2 \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 13 & 14 & 18 & 0 & 200 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$-13R_1 + R_2 \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & -13 & 200 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} -R_2 + R_1 \longrightarrow \\ \\ -2R_2 + R_3 \longrightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 14 & -200 & 0 \\ 0 & 1 & 5 & -13 & 200 & 0 \\ 0 & 0 & -11 & 26 & -400 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -\frac{1}{11}R_3 \longrightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 14 & -200 & 0 \\ 0 & 1 & 5 & -13 & 200 & 0 \\ 0 & 0 & 1 & -\frac{26}{11} & \frac{400}{11} & -\frac{1}{11} \end{array}\right]$$

$$\begin{array}{r} 4R_3 + R_1 \longrightarrow \\ -5R_3 + R_2 \longrightarrow \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{50}{11} & -\frac{600}{11} & -\frac{4}{11} \\ 0 & 1 & 0 & -\frac{13}{11} & \frac{200}{11} & \frac{5}{11} \\ 0 & 0 & 1 & -\frac{26}{11} & \frac{400}{11} & -\frac{1}{11} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix} \begin{bmatrix} 10{,}000 \\ 705 \\ 0 \end{bmatrix} = \begin{bmatrix} 7000 \\ 1000 \\ 2000 \end{bmatrix}$$

Solution: \$7000 in AAA-rated bonds, \$1000 in A-rated bonds, \$2000 in B-rated bonds

68. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0.065 & 0.07 & 0.09 \\ 0 & 2 & -1 \end{bmatrix}$

$$[A \ \vdots\ I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0.065 & 0.07 & 0.09 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$200R_2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 13 & 14 & 18 & 0 & 200 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$-13R_1 + R_2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & -13 & 200 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{matrix} -R_2 + R_1 \rightarrow \\ \\ -2R_2 + R_3 \rightarrow \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 14 & -200 & 0 \\ 0 & 1 & 5 & -13 & 200 & 0 \\ 0 & 0 & -11 & 26 & -400 & 1 \end{array}\right]$$

$$\begin{matrix} \\ \\ -\frac{1}{11}R_3 \rightarrow \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 14 & -200 & 0 \\ 0 & 1 & 5 & -13 & 200 & 0 \\ 0 & 0 & 1 & -\frac{26}{11} & \frac{400}{11} & -\frac{1}{11} \end{array}\right]$$

$$\begin{matrix} 4R_3 + R_1 \rightarrow \\ -5R_3 + R_2 \rightarrow \\ \\ \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{50}{11} & -\frac{600}{11} & -\frac{4}{11} \\ 0 & 1 & 0 & -\frac{13}{11} & \frac{200}{11} & \frac{5}{11} \\ 0 & 0 & 1 & -\frac{26}{11} & \frac{400}{11} & -\frac{1}{11} \end{array}\right] = [I \ \vdots\ A^{-1}]$$

$$X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix}\begin{bmatrix} 10{,}000 \\ 760 \\ 0 \end{bmatrix} = \begin{bmatrix} 4000 \\ 2000 \\ 4000 \end{bmatrix}$$

Solution: \$4000 in AAA-rated bonds, \$2000 in A-rated bonds, \$4000 in B-rated bonds.

69. Use the inverse matrix A^{-1} from Exercise 67.

$$X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix}\begin{bmatrix} 12{,}000 \\ 835 \\ 0 \end{bmatrix} = \begin{bmatrix} 9000 \\ 1000 \\ 2000 \end{bmatrix}$$

Solution: \$9000 in AAA-rated bonds, \$1000 in A-rated bonds, \$2000 in B-rated bonds

70. Use the inverse matrix A^{-1} from Exercise 69.

$$X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 50 & -600 & -4 \\ -13 & 200 & 5 \\ -26 & 400 & -1 \end{bmatrix}\begin{bmatrix} 500{,}000 \\ 38{,}000 \\ 0 \end{bmatrix} = \begin{bmatrix} 200{,}000 \\ 100{,}000 \\ 200{,}000 \end{bmatrix}$$

Solution: \$200,000 in AAA-rated bonds, \$100,000 in A-rated bonds, and \$200,000 in B-rated bonds.

71. (a) $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{bmatrix}$

$$[A \ \vdots \ I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} R_1 \\ \\ R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 2 & 0 & 4 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -2R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & -2 & 6 & 1 & 0 & -2 \end{array}\right]$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 0 & -1 & 1 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 14 & 1 & 2 & -2 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{14}R_3 \rightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 0 & -1 & 1 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{14} & \frac{1}{7} & -\frac{1}{7} \end{array}\right]$$

$$\begin{array}{r} 5R_3 + R_1 \rightarrow \\ -4R_3 + R_2 \rightarrow \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{5}{14} & -\frac{2}{7} & \frac{2}{7} \\ 0 & 1 & 0 & -\frac{2}{7} & \frac{3}{7} & \frac{4}{7} \\ 0 & 0 & 1 & \frac{1}{14} & \frac{1}{7} & -\frac{1}{7} \end{array}\right] = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \frac{1}{14}\begin{bmatrix} 5 & -4 & 4 \\ -4 & 6 & 8 \\ 1 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 5 & -4 & 4 \\ -4 & 6 & 8 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} 14 \\ 28 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ 5 \end{bmatrix}$$

Solution: $I_1 = -3$ amperes, $I_2 = 8$ amperes, $I_3 = 5$ amperes

(b) $$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 5 & -4 & 4 \\ -4 & 6 & 8 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} 24 \\ 23 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Solution:

$I_1 = 2$ amperes, $I_2 = 3$ amperes, $I_3 = 5$ amperes

72. (a) $n = 3; \sum_{i=1}^{n} x_i = 7 + 9 + 11 = 27;$

$$\sum_{i=1}^{n} y_i = 182.7 + 187.2 + 191.3 = 561.2;$$

$$\sum_{i=1}^{n} x_i^2 = 49 + 81 + 121 = 251$$

$$\sum_{i=1}^{n} x_i y_i = 7(182.7) + 9(187.2) + 11(191.3) = 5068$$

System: $\begin{cases} 3b + 27a = 561.2 \\ 27b + 251a = 5068 \end{cases}$

(b) $$\begin{bmatrix} 3 & 27 \\ 27 & 251 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{251}{24} & -\frac{9}{8} \\ -\frac{9}{8} & \frac{1}{8} \end{bmatrix}; \quad \begin{bmatrix} \frac{251}{24} & -\frac{9}{8} \\ -\frac{9}{8} & \frac{1}{8} \end{bmatrix}\begin{bmatrix} 561.2 \\ 5068 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{251}{24}\right)(561.2) + \left(-\frac{9}{8}\right)(5068) \\ \left(-\frac{9}{8}\right)(561.2) + \left(\frac{1}{8}\right)(5068) \end{bmatrix} = \begin{bmatrix} 167.7 \\ 2.15 \end{bmatrix}$$

$b = 167.7, a = 2.15$

The least squares regression line is $y = 2.15t + 167.7$.

(c) For 2003, $t = 13$; $y = 2.15(13) + 167.7 = 195.65$.

This projects about 196 million licensed drivers in 2003.

(d) The projected value is very close to the actual value.

(e) $2.15t + 167.7 = 208$

$$2.15t = 40.3$$

$$t \approx 18.7$$

Since $t = 18$ represents 2008, the model projects that the number of licensed drivers will reach 208 million during 2008.

73. True. If B is the inverse of A, then $AB = I = BA$.

74. True. If A and B are both square matrices and $AB = I_n$, it can be shown that $BA = I_n$.

75. $AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left(\dfrac{1}{ad-bc}\right)\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \dfrac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$= \frac{1}{ad-bc}\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

76. (a) Given $A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \dfrac{1}{a_{11}} & 0 \\ 0 & \dfrac{1}{a_{22}} \end{bmatrix}$.

Given $A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \dfrac{1}{a_{11}} & 0 & 0 \\ 0 & \dfrac{1}{a_{22}} & 0 \\ 0 & 0 & \dfrac{1}{a_{33}} \end{bmatrix}$.

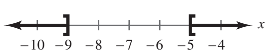
(b) In general, the inverse of a matrix in the form of A is

$$\begin{bmatrix} \dfrac{1}{a_{11}} & 0 & 0 & \cdots & 0 \\ 0 & \dfrac{1}{a_{22}} & 0 & \cdots & 0 \\ 0 & 0 & \dfrac{1}{a_{33}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \dfrac{1}{a_{nn}} \end{bmatrix}$$

77. $|x + 7| \ge 2$

$x + 7 \le -2$ or $x + 7 \ge 2$

$x \le -9$ or $x \ge -5$

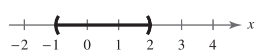


78. $|2x - 1| < 3$

$-3 < 2x - 1 < 3$

$-2 < 2x < 4$

$-1 < x < 2$



79. $3^{x/2} = 315$

$\ln 3^{x/2} = \ln 315$

$\dfrac{x}{2}\ln 3 = \ln 315$

$x = \dfrac{2\ln 315}{\ln 3} \approx 10.472$

80. $2000e^{-x/5} = 400$

$e^{-x/5} = \dfrac{1}{5}$

$\ln e^{-x/5} = \ln\dfrac{1}{5}$

$-\dfrac{x}{5} = \ln\dfrac{1}{5}$

$x = -5\ln\dfrac{1}{5} \approx -8.047$

81. $\log_2 x - 2 = 4.5$

$\log_2 x = 6.5$

$x = 2^{6.5} \approx 90.510$

82. $\ln x + \ln(x-1) = 0$

$\ln[x(x-1)] = 0$

$e^{\ln[x(x-1)]} = e^0$

$x(x-1) = 1$

$x^2 - x - 1 = 0$

$x = \dfrac{1 \pm \sqrt{1 - 4(-1)}}{2}$

$x = \dfrac{1 \pm \sqrt{5}}{2}$

Choose the positive value only:

$x = \dfrac{1 + \sqrt{5}}{2} \approx 1.618$

83. Answers will vary.

Section 8.4 The Determinant of a Square Matrix

- You should be able to determine the determinant of a matrix of order 2×2 by using the difference of the products of the diagonals.
- You should be able to use expansion by cofactors to find the determinant of a matrix of order 3×3 or greater.
- The determinant of a triangular matrix equals the product of the entries on the main diagonal.

Vocabulary Check

1. determinant **2.** minor **3.** cofactor **4.** expanding by cofactors

1. 5

2. -8

3. $\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 2(4) - 1(3) = 8 - 3 = 5$

4. $\begin{vmatrix} -3 & 1 \\ 5 & 2 \end{vmatrix} = (-3)(2) - (5)(1) = -11$

5. $\begin{vmatrix} 5 & 2 \\ -6 & 3 \end{vmatrix} = 5(3) - 2(-6) = 15 + 12 = 27$

6. $\begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = (2)(3) - (4)(-2) = 14$

7. $\begin{vmatrix} -7 & 0 \\ 3 & 0 \end{vmatrix} = -7(0) - 0(3) = 0$

8. $\begin{vmatrix} 4 & -3 \\ 0 & 0 \end{vmatrix} = (4)(0) - (0)(-3) = 0$

9. $\begin{vmatrix} 2 & 6 \\ 0 & 3 \end{vmatrix} = 2(3) - 6(0) = 6$

10. $\begin{vmatrix} 2 & -3 \\ -6 & 9 \end{vmatrix} = (2)(9) - (-6)(-3) = 0$

11. $\begin{vmatrix} -3 & -2 \\ -6 & -1 \end{vmatrix} = (-3)(-1) - (-2)(-6) = 3 - 12 = -9$

12. $\begin{vmatrix} 4 & 7 \\ -2 & 5 \end{vmatrix} = (4)(5) - (-2)(7) = 34$

13. $\begin{vmatrix} 9 & 0 \\ 7 & 8 \end{vmatrix} = 9(8) - 0(7) = 72 - 0 = 72$

14. $\begin{vmatrix} 0 & 6 \\ -3 & 2 \end{vmatrix} = (0)(2) - (-3)(6) = 18$

15. $\begin{vmatrix} -\frac{1}{2} & \frac{1}{3} \\ -6 & \frac{1}{3} \end{vmatrix} = -\frac{1}{2}\left(\frac{1}{3}\right) - \frac{1}{3}(-6) = -\frac{1}{6} + 2 = \frac{11}{6}$

16. $\begin{vmatrix} \frac{2}{3} & \frac{4}{3} \\ -1 & -\frac{1}{3} \end{vmatrix} = \left(\frac{2}{3}\right)\left(-\frac{1}{3}\right) - (-1)\left(\frac{4}{3}\right) = \frac{10}{9}$

17. $\begin{vmatrix} 0.3 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ -0.4 & 0.4 & 0.3 \end{vmatrix} = -0.002$

18. $\begin{vmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{vmatrix} = -0.022$

19. $\begin{vmatrix} 0.9 & 0.7 & 0 \\ -0.1 & 0.3 & 1.3 \\ -2.2 & 4.2 & 6.1 \end{vmatrix} = -4.842$

20. $\begin{vmatrix} 0.1 & 0.1 & -4.3 \\ 7.5 & 6.2 & 0.7 \\ 0.3 & 0.6 & -1.2 \end{vmatrix} = -11.217$

21. $\begin{vmatrix} 1 & 4 & -2 \\ 3 & 6 & -6 \\ -2 & 1 & 4 \end{vmatrix} = 0$

22. $\begin{vmatrix} 2 & 3 & 1 \\ 0 & 5 & -2 \\ 0 & 0 & -2 \end{vmatrix} = -20$

23. $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}$

(a) $M_{11} = -5$
$M_{12} = 2$
$M_{21} = 4$
$M_{22} = 3$

(b) $C_{11} = M_{11} = -5$
$C_{12} = -M_{12} = -2$
$C_{21} = -M_{21} = -4$
$C_{22} = M_{22} = 3$

24. $\begin{bmatrix} 11 & 0 \\ -3 & 2 \end{bmatrix}$

(a) $M_{11} = 2$
$M_{12} = -3$
$M_{21} = 0$
$M_{22} = 11$

(b) $C_{11} = M_{11} = 2$
$C_{12} = -M_{12} = 3$
$C_{21} = M_{21} = 0$
$C_{22} = M_{22} = 11$

25. $\begin{bmatrix} 3 & 1 \\ -2 & -4 \end{bmatrix}$

(a) $M_{11} = -4$
$M_{12} = -2$
$M_{21} = 1$
$M_{22} = 3$

(b) $C_{11} = M_{11} = -4$
$C_{12} = -M_{12} = 2$
$C_{21} = -M_{21} = -1$
$C_{22} = M_{22} = 3$

26. $\begin{bmatrix} -6 & 5 \\ 7 & -2 \end{bmatrix}$

(a) $M_{11} = -2$
$M_{12} = 7$
$M_{21} = 5$
$M_{22} = -6$

(b) $C_{11} = M_{11} = -2$
$C_{12} = -M_{12} = -7$
$C_{21} = -M_{21} = -5$
$C_{22} = M_{22} = -6$

27. $\begin{bmatrix} 4 & 0 & 2 \\ -3 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 2 - (-1) = 3$

$M_{12} = \begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix} = -3 - 1 = -4$

$M_{13} = \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix} = 3 - 2 = 1$

$M_{21} = \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 0 - (-2) = 2$

$M_{22} = \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = 4 - 2 = 2$

$M_{23} = \begin{vmatrix} 4 & 0 \\ 1 & -1 \end{vmatrix} = -4 - 0 = -4$

$M_{31} = \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 0 - 4 = -4$

$M_{32} = \begin{vmatrix} 4 & 2 \\ -3 & 1 \end{vmatrix} = 4 - (-6) = 10$

$M_{33} = \begin{vmatrix} 4 & 0 \\ -3 & 2 \end{vmatrix} = 8 - 0 = 8$

(b) $C_{11} = (-1)^2 M_{11} = 3$
$C_{12} = (-1)^3 M_{12} = 4$
$C_{13} = (-1)^4 M_{13} = 1$
$C_{21} = (-1)^3 M_{21} = -2$
$C_{22} = (-1)^4 M_{22} = 2$
$C_{23} = (-1)^5 M_{23} = 4$
$C_{31} = (-1)^4 M_{31} = -4$
$C_{32} = (-1)^5 M_{32} = -10$
$C_{33} = (-1)^6 M_{33} = 8$

28. $\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & 5 \\ 4 & -6 & 4 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 2 & 5 \\ -6 & 4 \end{vmatrix} = 8-(-30) = 38$

$M_{12} = \begin{vmatrix} 3 & 5 \\ 4 & 4 \end{vmatrix} = 12 - 20 = -8$

$M_{13} = \begin{vmatrix} 3 & 2 \\ 4 & -6 \end{vmatrix} = -18 - 8 = -26$

$M_{21} = \begin{vmatrix} -1 & 0 \\ -6 & 4 \end{vmatrix} = -4 - 0 = -4$

$M_{22} = \begin{vmatrix} 1 & 0 \\ 4 & 4 \end{vmatrix} = 4 - 0 = 4$

$M_{23} = \begin{vmatrix} 1 & -1 \\ 4 & -6 \end{vmatrix} = -6 - (-4) = -2$

$M_{31} = \begin{vmatrix} -1 & 0 \\ 2 & 5 \end{vmatrix} = -5 - 0 = -5$

$M_{32} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

$M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 2 - (-3) = 5$

(b) $C_{11} = (-1)^2 M_{11} = 38$
$C_{12} = (-1)^3 M_{12} = 8$
$C_{13} = (-1)^4 M_{13} = -26$
$C_{21} = (-1)^3 M_{21} = 4$
$C_{22} = (-1)^4 M_{22} = 4$
$C_{23} = (-1)^5 M_{23} = 2$
$C_{31} = (-1)^4 M_{31} = -5$
$C_{32} = (-1)^5 M_{32} = -5$
$C_{33} = (-1)^6 M_{33} = 5$

29. $\begin{bmatrix} 3 & -2 & 8 \\ 3 & 2 & -6 \\ -1 & 3 & 6 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 2 & -6 \\ 3 & 6 \end{vmatrix} = 12 + 18 = 30$

$M_{12} = \begin{vmatrix} 3 & -6 \\ -1 & 6 \end{vmatrix} = 18 - 6 = 12$

$M_{13} = \begin{vmatrix} 3 & 2 \\ -1 & 3 \end{vmatrix} = 9 + 2 = 11$

$M_{21} = \begin{vmatrix} -2 & 8 \\ 3 & 6 \end{vmatrix} = -12 - 24 = -36$

$M_{22} = \begin{vmatrix} 3 & 8 \\ -1 & 6 \end{vmatrix} = 18 + 8 = 26$

$M_{23} = \begin{vmatrix} 3 & -2 \\ -1 & 3 \end{vmatrix} = 9 - 2 = 7$

$M_{31} = \begin{vmatrix} -2 & 8 \\ 2 & -6 \end{vmatrix} = 12 - 16 = -4$

$M_{32} = \begin{vmatrix} 3 & 8 \\ 3 & -6 \end{vmatrix} = -18 - 24 = -42$

$M_{33} = \begin{vmatrix} 3 & -2 \\ 3 & 2 \end{vmatrix} = 6 + 6 = 12$

(b) $C_{11} = (-1)^2 M_{11} = 30$

$C_{12} = (-1)^3 M_{12} = -12$

$C_{13} = (-1)^4 M_{13} = 11$

$C_{21} = (-1)^3 M_{21} = 36$

$C_{22} = (-1)^4 M_{22} = 26$

$C_{23} = (-1)^5 M_{23} = -7$

$C_{31} = (-1)^4 M_{31} = -4$

$C_{32} = (-1)^5 M_{32} = 42$

$C_{33} = (-1)^6 M_{33} = 12$

30. $\begin{bmatrix} -2 & 9 & 4 \\ 7 & -6 & 0 \\ 6 & 7 & -6 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} -6 & 0 \\ 7 & -6 \end{vmatrix} = 36$

$M_{12} = \begin{vmatrix} 7 & 0 \\ 6 & -6 \end{vmatrix} = -42$

$M_{13} = \begin{vmatrix} 7 & -6 \\ 6 & 7 \end{vmatrix} = 85$

$M_{21} = \begin{vmatrix} 9 & 4 \\ 7 & -6 \end{vmatrix} = -82$

$M_{22} = \begin{vmatrix} -2 & 4 \\ 6 & -6 \end{vmatrix} = -12$

$M_{23} = \begin{vmatrix} -2 & 9 \\ 6 & 7 \end{vmatrix} = -68$

$M_{31} = \begin{vmatrix} 9 & 4 \\ -6 & 0 \end{vmatrix} = 24$

$M_{32} = \begin{vmatrix} -2 & 4 \\ 7 & 0 \end{vmatrix} = -28$

$M_{33} = \begin{vmatrix} -2 & 9 \\ 7 & -6 \end{vmatrix} = -51$

(b) $C_{11} = (-1)^2 M_{11} = 36$

$C_{12} = (-1)^3 M_{12} = 42$

$C_{13} = (-1)^4 M_{13} = 85$

$C_{21} = (-1)^3 M_{21} = 82$

$C_{22} = (-1)^4 M_{22} = -12$

$C_{23} = (-1)^5 M_{23} = 68$

$C_{31} = (-1)^4 M_{31} = 24$

$C_{32} = (-1)^5 M_{32} = 28$

$C_{33} = (-1)^6 M_{33} = -51$

31. (a) $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix} = -3\begin{vmatrix} 5 & 6 \\ -3 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 5 \\ 2 & -3 \end{vmatrix} = -3(23) - 2(-8) - 22 = -75$

(b) $\begin{vmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{vmatrix} = -2\begin{vmatrix} 4 & 6 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} -3 & 1 \\ 4 & 6 \end{vmatrix} = -2(-8) + 5(-5) + 3(-22) = -75$

32. (a) $\begin{vmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{vmatrix} = -6\begin{vmatrix} 4 & 2 \\ -7 & -8 \end{vmatrix} + 3\begin{vmatrix} -3 & 2 \\ 4 & -8 \end{vmatrix} - 1\begin{vmatrix} -3 & 4 \\ 4 & -7 \end{vmatrix} = -6(-18) + 3(16) - (5) = 151$

(b) $\begin{vmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{vmatrix} = 2\begin{vmatrix} 6 & 3 \\ 4 & -7 \end{vmatrix} - \begin{vmatrix} -3 & 4 \\ 4 & -7 \end{vmatrix} - 8\begin{vmatrix} -3 & 4 \\ 6 & 3 \end{vmatrix} = 2(-54) - (5) - 8(-33) = 151$

33. (a) $\begin{vmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{vmatrix} = 0\begin{vmatrix} 0 & -3 \\ 6 & 3 \end{vmatrix} + 12\begin{vmatrix} 5 & -3 \\ 1 & 3 \end{vmatrix} - 4\begin{vmatrix} 5 & 0 \\ 1 & 6 \end{vmatrix} = 0(18) + 12(18) - 4(30) = 96$

(b) $\begin{vmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{vmatrix} = 0\begin{vmatrix} 0 & 4 \\ 1 & 3 \end{vmatrix} + 12\begin{vmatrix} 5 & -3 \\ 1 & 3 \end{vmatrix} - 6\begin{vmatrix} 5 & -3 \\ 0 & 4 \end{vmatrix} = 0(-4) + 12(18) - 6(20) = 96$

34. (a) $\begin{vmatrix} 10 & -5 & 5 \\ 30 & 0 & 10 \\ 0 & 10 & 1 \end{vmatrix} = 0\begin{vmatrix} -5 & 5 \\ 0 & 10 \end{vmatrix} - 10\begin{vmatrix} 10 & 5 \\ 30 & 10 \end{vmatrix} + \begin{vmatrix} 10 & -5 \\ 30 & 0 \end{vmatrix} = 0(-50) - 10(-50) + 150 = 650$

(b) $\begin{vmatrix} 10 & -5 & 5 \\ 30 & 0 & 10 \\ 0 & 10 & 1 \end{vmatrix} = 10\begin{vmatrix} 0 & 10 \\ 10 & 1 \end{vmatrix} - 30\begin{vmatrix} -5 & 5 \\ 10 & 1 \end{vmatrix} + 0\begin{vmatrix} -5 & 5 \\ 0 & 10 \end{vmatrix} = 10(-100) - 30(-55) + 0(-50) = 650$

35. (a) $\begin{vmatrix} 6 & 0 & -3 & 5 \\ 4 & 13 & 6 & -8 \\ -1 & 0 & 7 & 4 \\ 8 & 6 & 0 & 2 \end{vmatrix} = -4\begin{vmatrix} 0 & -3 & 5 \\ 0 & 7 & 4 \\ 6 & 0 & 2 \end{vmatrix} + 13\begin{vmatrix} 6 & -3 & 5 \\ -1 & 7 & 4 \\ 8 & 0 & 2 \end{vmatrix} - 6\begin{vmatrix} 6 & 0 & 5 \\ -1 & 0 & 4 \\ 8 & 6 & 2 \end{vmatrix} - 8\begin{vmatrix} 6 & 0 & -3 \\ -1 & 0 & 7 \\ 8 & 6 & 0 \end{vmatrix}$

$= -4(-282) + 13(-298) - 6(-174) - 8(-234) = 170$

(b) $\begin{vmatrix} 6 & 0 & -3 & 5 \\ 4 & 13 & 6 & -8 \\ -1 & 0 & 7 & 4 \\ 8 & 6 & 0 & 2 \end{vmatrix} = 0\begin{vmatrix} 4 & 6 & -8 \\ -1 & 7 & 4 \\ 8 & 0 & 2 \end{vmatrix} + 13\begin{vmatrix} 6 & -3 & 5 \\ -1 & 7 & 4 \\ 8 & 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 6 & -3 & 5 \\ 4 & 6 & -8 \\ 8 & 0 & 2 \end{vmatrix} + 6\begin{vmatrix} 6 & -3 & 5 \\ 4 & 6 & -8 \\ -1 & 7 & 4 \end{vmatrix}$

$= 0 + 13(-298) + 0 + 6(674) = 170$

36. (a) $\begin{vmatrix} 10 & 8 & 3 & -7 \\ 4 & 0 & 5 & -6 \\ 0 & 3 & 2 & 7 \\ 1 & 0 & -3 & 2 \end{vmatrix} = 0\begin{vmatrix} 8 & 3 & -7 \\ 0 & 5 & -6 \\ 0 & -3 & 2 \end{vmatrix} - 3\begin{vmatrix} 10 & 3 & -7 \\ 4 & 5 & -6 \\ 1 & -3 & 2 \end{vmatrix} + 2\begin{vmatrix} 10 & 8 & -7 \\ 4 & 0 & -6 \\ 1 & 0 & 2 \end{vmatrix} - 7\begin{vmatrix} 10 & 8 & 3 \\ 4 & 0 & 5 \\ 1 & 0 & -3 \end{vmatrix}$

$= 0(-64) - 3(-3) + 2(-112) - 7(136) = -1167$

(b) $\begin{vmatrix} 10 & 8 & 3 & -7 \\ 4 & 0 & 5 & -6 \\ 0 & 3 & 2 & 7 \\ 1 & 0 & -3 & 2 \end{vmatrix} = 10\begin{vmatrix} 0 & 5 & -6 \\ 3 & 2 & 7 \\ 0 & -3 & 2 \end{vmatrix} - 4\begin{vmatrix} 8 & 3 & -7 \\ 3 & 2 & 7 \\ 0 & -3 & 2 \end{vmatrix} + 0\begin{vmatrix} 8 & 3 & -7 \\ 0 & 5 & -6 \\ 0 & -3 & 2 \end{vmatrix} - 1\begin{vmatrix} 8 & 3 & -7 \\ 0 & 5 & -6 \\ 3 & 2 & 7 \end{vmatrix}$

$= 10(24) - 4(245) + 0(-64) - 1(427) = -1167$

37. Expand along Column 1.

$\begin{vmatrix} 2 & -1 & 0 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} + 4\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = 2(0) - 4(-1) + 4(-1) = 0$

38. Expand along Row 3.

$\begin{vmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 0\begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} -2 & 3 \\ 1 & 0 \end{vmatrix} + 4\begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix}$

$= 0(3) - 1(-3) + 4(0) = 3$

39. Expand along Row 2.

$$\begin{vmatrix} 6 & 3 & -7 \\ 0 & 0 & 0 \\ 4 & -6 & 3 \end{vmatrix} = 0\begin{vmatrix} 3 & -7 \\ -6 & 3 \end{vmatrix} - 0\begin{vmatrix} 6 & -7 \\ 4 & 3 \end{vmatrix} + 0\begin{vmatrix} 6 & 3 \\ 4 & -6 \end{vmatrix} = 0$$

40. Expand along Column 3.

$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{vmatrix} = 2\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= 2(2) - 0(2) + 3(-2) = -2$$

41. $\begin{vmatrix} -1 & 2 & -5 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{vmatrix} = (-1)(3)(3) = -9$ (Upper triangular)

42. Expand along Row 1.

$$\begin{vmatrix} 1 & 0 & 0 \\ -4 & -1 & 0 \\ 5 & 1 & 5 \end{vmatrix} = 1\begin{vmatrix} -1 & 0 \\ 1 & 5 \end{vmatrix} - 0\begin{vmatrix} -4 & 0 \\ 5 & 5 \end{vmatrix} + 0\begin{vmatrix} -4 & -1 \\ 5 & 1 \end{vmatrix}$$

$$= 1(-5) - 0(-20) + 0(1) = -5$$

43. Expand along Column 3.

$$\begin{vmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{vmatrix} = -2\begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}$$

$$= -2(14) + 3(-10) = -58$$

44. Expand along Row 3.

$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & 4 \\ 1 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} -1 & 3 \\ 4 & 4 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix} = 1(-16) - 0(5) + 2(9) = 2$$

45. $\begin{vmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{vmatrix} = (2)(3)(-5) = -30$ (Upper triangular)

46. Expand along Row 1.

$$\begin{vmatrix} -3 & 0 & 0 \\ 7 & 11 & 0 \\ 1 & 2 & 2 \end{vmatrix} = -3\begin{vmatrix} 11 & 0 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 7 & 0 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 7 & 11 \\ 1 & 2 \end{vmatrix}$$

$$= -3(22) - 0(14) + 0(3) = -66$$

47. Expand along Column 3.

$$\begin{vmatrix} 2 & 6 & 6 & 2 \\ 2 & 7 & 3 & 6 \\ 1 & 5 & 0 & 1 \\ 3 & 7 & 0 & 7 \end{vmatrix} = 6\begin{vmatrix} 2 & 7 & 6 \\ 1 & 5 & 1 \\ 3 & 7 & 7 \end{vmatrix} - 3\begin{vmatrix} 2 & 6 & 2 \\ 1 & 5 & 1 \\ 3 & 7 & 7 \end{vmatrix} = 6(-20) - 3(16) = -168$$

48. Expand along Row 2.

$$\begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix} = -(-2)\begin{vmatrix} 6 & -5 & 4 \\ 1 & 2 & 2 \\ 3 & -1 & -1 \end{vmatrix} - 6\begin{vmatrix} 3 & 6 & 4 \\ 1 & 1 & 2 \\ 0 & 3 & -1 \end{vmatrix} = 2(-63) - 6(-3) = -108$$

49. Expand along Column 1.

$$\begin{vmatrix} 5 & 3 & 0 & 6 \\ 4 & 6 & 4 & 12 \\ 0 & 2 & -3 & 4 \\ 0 & 1 & -2 & 2 \end{vmatrix} = 5\begin{vmatrix} 6 & 4 & 12 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} - 4\begin{vmatrix} 3 & 0 & 6 \\ 2 & -3 & 4 \\ 1 & -2 & 2 \end{vmatrix} = 5(0) - 4(0) = 0$$

50. Expand along Row 3.

$$\begin{vmatrix} 1 & 4 & 3 & 2 \\ -5 & 6 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & -2 & 1 & 5 \end{vmatrix} = 0$$

51. Expand along Column 2, then along Column 4.

$$\begin{vmatrix} 3 & 2 & 4 & -1 & 5 \\ -2 & 0 & 1 & 3 & 2 \\ 1 & 0 & 0 & 4 & 0 \\ 6 & 0 & 2 & -1 & 0 \\ 3 & 0 & 5 & 1 & 0 \end{vmatrix} = -2\begin{vmatrix} -2 & 1 & 3 & 2 \\ 1 & 0 & 4 & 0 \\ 6 & 2 & -1 & 0 \\ 3 & 5 & 1 & 0 \end{vmatrix} = (-2)(-2)\begin{vmatrix} 1 & 0 & 4 \\ 6 & 2 & -1 \\ 3 & 5 & 1 \end{vmatrix} = 4(103) = 412$$

52. Expand along Column 1.

$$\begin{vmatrix} 5 & 2 & 0 & 0 & -2 \\ 0 & 1 & 4 & 3 & 2 \\ 0 & 0 & 2 & 6 & 3 \\ 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix} = 5\begin{vmatrix} 1 & 4 & 3 & 2 \\ 0 & 2 & 6 & 3 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 5 \cdot 1\begin{vmatrix} 2 & 6 & 3 \\ 3 & 4 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 5(-20) = -100$$

53. $\begin{vmatrix} 3 & 8 & -7 \\ 0 & -5 & 4 \\ 8 & 1 & 6 \end{vmatrix} = -126$

54. $\begin{vmatrix} 5 & -8 & 0 \\ 9 & 7 & 4 \\ -8 & 7 & 1 \end{vmatrix} = 223$

55. $\begin{vmatrix} 7 & 0 & -14 \\ -2 & 5 & 4 \\ -6 & 2 & 12 \end{vmatrix} = 0$

56. $\begin{vmatrix} 3 & 0 & 0 \\ -2 & 5 & 0 \\ 12 & 5 & 7 \end{vmatrix} = 105$

57. $\begin{vmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & -4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{vmatrix} = -336$

58. $\begin{vmatrix} 0 & -3 & 8 & 2 \\ 8 & 1 & -1 & 6 \\ -4 & 6 & 0 & 9 \\ -7 & 0 & 0 & 14 \end{vmatrix} = 7441$

59. $\begin{vmatrix} 3 & -2 & 4 & 3 & 1 \\ -1 & 0 & 2 & 1 & 0 \\ 5 & -1 & 0 & 3 & 2 \\ 4 & 7 & -8 & 0 & 0 \\ 1 & 2 & 3 & 0 & 2 \end{vmatrix} = 410$

60. $\begin{vmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{vmatrix} = -48$

61. (a) $\begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} = -3$

(b) $\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$

(d) $\begin{vmatrix} -2 & 0 \\ 0 & -3 \end{vmatrix} = 6$

62. (a) $|A| = \begin{vmatrix} -2 & 1 \\ 4 & -2 \end{vmatrix} = 0$

(b) $|B| = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$

(c) $AB = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ 4 & 10 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} -2 & -5 \\ 4 & 10 \end{vmatrix} = 0$

63. (a) $\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} = -8$

(b) $\begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} = 0$

(c) $\begin{bmatrix} 4 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 1 & -1 \end{bmatrix}$

(d) $\begin{vmatrix} -4 & 4 \\ 1 & -1 \end{vmatrix} = 0$

64. (a) $|A| = \begin{vmatrix} 5 & 4 \\ 3 & -1 \end{vmatrix} = -17$

(b) $|B| = \begin{vmatrix} 0 & 6 \\ 1 & -2 \end{vmatrix} = -6$

(c) $AB = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 0 & 6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 22 \\ -1 & 20 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} 4 & 22 \\ -1 & 20 \end{vmatrix} = 102$

65. (a) $\begin{vmatrix} 0 & 1 & 2 \\ -3 & -2 & 1 \\ 0 & 4 & 1 \end{vmatrix} = -21$

(b) $\begin{vmatrix} 3 & -2 & 0 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = -19$

(c) $\begin{bmatrix} 0 & 1 & 2 \\ -3 & -2 & 1 \\ 0 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 & 0 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 1 & 4 \\ -8 & 9 & -3 \\ 7 & -3 & 9 \end{bmatrix}$

(d) $\begin{vmatrix} 7 & 1 & 4 \\ -8 & 9 & -3 \\ 7 & -3 & 9 \end{vmatrix} = 399$

66. (a) $|A| = \begin{vmatrix} 3 & 2 & 0 \\ -1 & -3 & 4 \\ -2 & 0 & 1 \end{vmatrix} = -23$

(b) $|B| = \begin{vmatrix} -3 & 0 & 1 \\ 0 & 2 & -1 \\ -2 & -1 & 1 \end{vmatrix} = 1$

(c) $AB = \begin{bmatrix} 3 & 2 & 0 \\ -1 & -3 & 4 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 & 1 \\ 0 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -9 & 4 & 1 \\ -5 & -10 & 6 \\ 4 & -1 & -1 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} -9 & 4 & 1 \\ -5 & -10 & 6 \\ 4 & -1 & -1 \end{vmatrix} = -23$

67. (a) $\begin{vmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 2$

(b) $\begin{vmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = -6$

(c) $\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}$

(d) $\begin{vmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{vmatrix} = -12$

68. (a) $|A| = \begin{vmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{vmatrix} = 0$

(b) $|B| = \begin{vmatrix} 2 & -1 & 4 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \end{vmatrix} = -7$

(c) $AB = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 4 \\ 0 & 1 & 3 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -4 & 9 \\ 8 & -6 & 3 \\ 6 & -2 & 15 \end{bmatrix}$

(d) $|AB| = \begin{vmatrix} 7 & -4 & 9 \\ 8 & -6 & 3 \\ 6 & -2 & 15 \end{vmatrix} = 0$

69. $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = wz - xy$

$-\begin{vmatrix} y & z \\ w & x \end{vmatrix} = -(xy - wz) = wz - xy$

Thus, $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = -\begin{vmatrix} y & z \\ w & x \end{vmatrix}$.

70. $\begin{vmatrix} w & cx \\ y & cz \end{vmatrix} = cwz - cxy = c(wz - xy)$

$c\begin{vmatrix} w & x \\ y & z \end{vmatrix} = c(wz - xy)$

So, $\begin{vmatrix} w & cx \\ y & cz \end{vmatrix} = c\begin{vmatrix} w & x \\ y & z \end{vmatrix}$.

71. $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = wz - xy$

$\begin{vmatrix} w & x + cw \\ y & z + cy \end{vmatrix} = w(z + cy) - y(x + cw) = wz - xy$

Thus, $\begin{vmatrix} w & x \\ y & z \end{vmatrix} = \begin{vmatrix} w & x + cw \\ y & z + cy \end{vmatrix}$.

72. $\begin{vmatrix} w & x \\ cw & cx \end{vmatrix} = cxw - cxw = 0$

So, $\begin{vmatrix} w & x \\ cw & cx \end{vmatrix} = 0$.

73.
$$\begin{aligned}
\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} &= \begin{vmatrix} y & y^2 \\ z & z^2 \end{vmatrix} - \begin{vmatrix} x & x^2 \\ z & z^2 \end{vmatrix} + \begin{vmatrix} x & x^2 \\ y & y^2 \end{vmatrix} \\
&= (yz^2 - y^2z) - (xz^2 - x^2z) + (xy^2 - x^2y) \\
&= yz^2 - xz^2 - y^2z + x^2z + xy(y - x) \\
&= z^2(y - x) - z(y^2 - x^2) + xy(y - x) \\
&= z^2(y - x) - z(y - x)(y + x) + xy(y - x) \\
&= (y - x)[z^2 - z(y + x) + xy] \\
&= (y - x)[z^2 - zy - zx + xy] \\
&= (y - x)[z^2 - zx - zy + xy] \\
&= (y - x)[z(z - x) - y(z - x)] \\
&= (y - x)(z - x)(z - y)
\end{aligned}$$

74.
$$\begin{aligned}
\begin{vmatrix} a+b & a & a \\ a & a+b & a \\ a & a & a+b \end{vmatrix} &= (a + b)\begin{vmatrix} a+b & a \\ a & a+b \end{vmatrix} - a\begin{vmatrix} a & a \\ a & a+b \end{vmatrix} + a\begin{vmatrix} a & a \\ a+b & a \end{vmatrix} \\
&= (a + b)[(a + b)^2 - a^2] - a[a(a + b) - a^2] + a[a^2 - a(a + b)] \\
&= (a + b)^3 - a^2(a + b) - a^2(a + b) + a^3 + a^3 - a^2(a + b) \\
&= (a + b)^3 - 3a^2(a + b) + 2a^3 \\
&= a^3 + 3a^2b + 3ab^2 + b^3 - 3a^3 - 3a^2b + 2a^3 \\
&= 3ab^2 + b^3 = b^2(3a + b)
\end{aligned}$$

75.
$$\begin{aligned}
\begin{vmatrix} x - 1 & 2 \\ 3 & x - 2 \end{vmatrix} &= 0 \\
(x - 1)(x - 2) - 6 &= 0 \\
x^2 - 3x - 4 &= 0 \\
(x + 1)(x - 4) &= 0 \\
x = -1 \text{ or } x &= 4
\end{aligned}$$

76.
$$\begin{aligned}
\begin{vmatrix} x - 2 & -1 \\ -3 & x \end{vmatrix} &= 0 \\
x(x - 2) - (-3)(-1) &= 0 \\
x^2 - 2x - 3 &= 0 \\
(x + 1)(x - 3) &= 0 \\
x = -1 \text{ or } x &= 3
\end{aligned}$$

77.
$$\begin{aligned}
\begin{vmatrix} x + 3 & 2 \\ 1 & x + 2 \end{vmatrix} &= 0 \\
(x + 3)(x + 2) - 2 &= 0 \\
x^2 + 5x + 4 &= 0 \\
(x + 1)(x + 4) &= 0 \\
x = -1 \text{ or } x &= -4
\end{aligned}$$

78.
$$\begin{aligned}
\begin{vmatrix} x + 4 & -2 \\ 7 & x - 5 \end{vmatrix} &= 0 \\
(x + 4)(x - 5) - 7(-2) &= 0 \\
x^2 - x - 6 &= 0 \\
(x + 2)(x - 3) &= 0 \\
x = -2 \text{ or } x &= 3
\end{aligned}$$

79. $\begin{vmatrix} 4u & -1 \\ -1 & 2v \end{vmatrix} = 8uv - 1$

80. $\begin{vmatrix} 3x^2 & -3y^2 \\ 1 & 1 \end{vmatrix} = 3x^2 - (-3y^2) = 3x^2 + 3y^2$

81. $\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} = 3e^{5x} - 2e^{5x} = e^{5x}$

82. $\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & (1-x)e^{-x} \end{vmatrix} = (1-x)e^{-2x} - (-xe^{-2x}) = e^{-2x} - xe^{-2x} + xe^{-2x} = e^{-2x}$

83. $\begin{vmatrix} x & \ln x \\ 1 & \dfrac{1}{x} \end{vmatrix} = 1 - \ln x$

84. $\begin{vmatrix} x & x \ln x \\ 1 & 1 + \ln x \end{vmatrix} = x(1 + \ln x) - x \ln x$

$= x + x \ln x - x \ln x = x$

85. True. If an entire row is zero, then each cofactor in the expansion is multiplied by zero.

86. True. If a square matrix has two columns that are equal, then elementary column operations can be used to create a column with all zeros.

87. Let $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 0 \\ 3 & 5 \end{bmatrix}$.

$|A| = \begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix} = 10,\ |B| = \begin{vmatrix} -4 & 0 \\ 3 & 5 \end{vmatrix} = -20,\ |A| + |B| = -10$

$A + B = \begin{bmatrix} -3 & 3 \\ 1 & 9 \end{bmatrix},\ |A + B| = \begin{vmatrix} -3 & 3 \\ 1 & 9 \end{vmatrix} = -30$

Thus, $|A + B| \neq |A| + |B|$. Your answer may differ, depending on how you choose A and B.

88. (a) $\begin{vmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{vmatrix} = 0$

$\begin{vmatrix} 33 & 34 & 35 \\ 36 & 37 & 38 \\ 39 & 40 & 41 \end{vmatrix} = 0$ $\qquad \begin{vmatrix} -5 & -4 & -3 \\ -2 & -1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$

$\begin{vmatrix} 19 & 20 & 21 & 22 \\ 23 & 24 & 25 & 26 \\ 27 & 28 & 29 & 30 \\ 31 & 32 & 33 & 34 \end{vmatrix} = 0$ $\qquad \begin{vmatrix} 57 & 58 & 59 & 60 \\ 61 & 62 & 63 & 64 \\ 65 & 66 & 67 & 68 \\ 69 & 70 & 71 & 72 \end{vmatrix} = 0$

For an $n \times n$ matrix $(n > 2)$ with consecutive integer entries, the determinant appears to be 0.

(b) $\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix} = x\begin{vmatrix} x+4 & x+5 \\ x+7 & x+8 \end{vmatrix} - (x+1)\begin{vmatrix} x+3 & x+5 \\ x+6 & x+8 \end{vmatrix} + (x+2)\begin{vmatrix} x+3 & x+4 \\ x+6 & x+7 \end{vmatrix}$

$$\begin{aligned} &= x[(x+4)(x+8) - (x+7)(x+5)] - (x+1)[(x+3)(x+8) \\ &\qquad - (x+6)(x+5)] + (x+2)[(x+3)(x+7) - (x+6)(x+4)] \\ &= x[(x^2 + 12x + 32) - (x^2 + 12x + 35)] - (x+1)[(x^2 + 11x + 24) \\ &\qquad - (x^2 + 11x + 30)] + (x+2)[(x^2 + 10x + 21) - (x^2 + 10x + 24)] \\ &= -3x - (x+1)(-6) + (x+2)(-3) \\ &= -3x + 6x + 6 - 3x - 6 = 0 \end{aligned}$$

89. A square matrix is a square array of numbers. The determinant of a square matrix is a real number.

90. Let $A = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ and $|A| = 5$.

$$2A = \begin{bmatrix} 2x_{11} & 2x_{12} & 2x_{13} \\ 2x_{21} & 2x_{22} & 2x_{23} \\ 2x_{31} & 2x_{32} & 2x_{33} \end{bmatrix}$$

$$\begin{aligned} |2A| &= 2x_{11}\begin{vmatrix} 2x_{22} & 2x_{23} \\ 2x_{32} & 2x_{33} \end{vmatrix} - 2x_{12}\begin{vmatrix} 2x_{21} & 2x_{23} \\ 2x_{31} & 2x_{33} \end{vmatrix} + 2x_{13}\begin{vmatrix} 2x_{21} & 2x_{22} \\ 2x_{31} & 2x_{32} \end{vmatrix} \\ &= 2[x_{11}(4x_{22}x_{33} - 4x_{32}x_{23}) - x_{12}(4x_{21}x_{33} - 4x_{31}x_{23}) + x_{13}(4x_{21}x_{32} - 4x_{31}x_{22})] \\ &= 8[x_{11}(x_{22}x_{33} - x_{32}x_{23}) - x_{12}(x_{21}x_{33} - x_{31}x_{23}) + x_{13}(x_{21}x_{32} - x_{31}x_{22})] \\ &= 8|A| \end{aligned}$$

So, $|2A| = 8|A| = 8(5) = 40$.

91. (a) $\begin{vmatrix} 1 & 3 & 4 \\ -7 & 2 & -5 \\ 6 & 1 & 2 \end{vmatrix} = -115$

$-\begin{vmatrix} 1 & 4 & 3 \\ -7 & -5 & 2 \\ 6 & 2 & 1 \end{vmatrix} = -115$

Column 2 and Column 3 were interchanged.

(b) $\begin{vmatrix} 1 & 3 & 4 \\ -2 & 2 & 0 \\ 1 & 6 & 2 \end{vmatrix} = -40$

$-\begin{vmatrix} 1 & 6 & 2 \\ -2 & 2 & 0 \\ 1 & 3 & 4 \end{vmatrix} = -40$

Row 1 and Row 3 were interchanged.

92. (a) Multiplying Row 1 of the matrix $\begin{bmatrix} 1 & -3 \\ 5 & 2 \end{bmatrix}$ by -5 and adding it to Row 2 gives the matrix $\begin{bmatrix} 1 & -3 \\ 0 & 17 \end{bmatrix}$.

$\begin{vmatrix} 1 & -3 \\ 5 & 2 \end{vmatrix} = 17 = \begin{vmatrix} 1 & -3 \\ 0 & 17 \end{vmatrix}$

(b) Multiplying Row 2 of the matrix $\begin{bmatrix} 5 & 4 & 2 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{bmatrix}$ by -2 and adding it to Row 1 gives the matrix $\begin{bmatrix} 1 & 10 & -6 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{bmatrix}$.

$\begin{vmatrix} 5 & 4 & 2 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{vmatrix} = -11 = \begin{vmatrix} 1 & 10 & -6 \\ 2 & -3 & 4 \\ 7 & 6 & 3 \end{vmatrix}$

93. (a) $A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 5 & 10 \\ 2 & -3 \end{bmatrix}$

$|B| = \begin{vmatrix} 5 & 10 \\ 2 & -3 \end{vmatrix} = -35$

$5|A| = 5\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = -35$

Row 1 was multiplied by 5.

$|B| = 5|A|$

(b) $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -3 & 2 \\ 7 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 8 & -3 \\ 3 & -12 & 6 \\ 7 & 4 & 9 \end{bmatrix}$

$|B| = \begin{bmatrix} 1 & 8 & -3 \\ 3 & -12 & 6 \\ 7 & 4 & 9 \end{bmatrix} = -300$

$12|A| = 12\begin{vmatrix} 1 & 2 & -1 \\ 3 & -3 & 2 \\ 7 & 1 & 3 \end{vmatrix} = -300$

Column 2 was multiplied by 4 and Column 3 was multiplied by 3.

$|B| = (4)(3)|A| = 12|A|$

94. (a) $A = \begin{vmatrix} 7 & 0 \\ 0 & 4 \end{vmatrix}$, $|A| = 7(4) - 0 = 28$

(b) $A = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{vmatrix}$, $|A| = (-1)(5)(2) = -10$

(c) $A = \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{vmatrix}$

Using cofactors and a_{11}, $|A| = 2 \cdot C_{11} + 0 \cdot C_{12} + 0 \cdot C_{13} + 0 \cdot C_{14}$.

$C_{11} = \begin{vmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix}$

$|A| = 2C_{11} = 2(-2 \cdot 1 \cdot 3) = 2 \cdot (-6) = -12$

In each case, the determinant of the matrix is the product of the diagonal entries. From this, one would conjecture that the determinant of a diagonal matrix is the product of the diagonal entries.

95. $f(x) = x^3 - 2x$

Since f is a polynomial, the domain is all real numbers x.

96. $g(x) = \sqrt[3]{x}$

An odd root of a number is defined for all real numbers.
Domain: all real numbers x

97. $h(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0$

$(4 + x)(4 - x) \geq 0$

Critical numbers: $x = \pm 4$

Test intervals: $(-\infty, -4), (-4, 4), (4, \infty)$

Test: Is $16 - x^2 \geq 0$?

Solution: $[-4, 4]$

Domain of h: $-4 \leq x \leq 4$

98. $A(x) = \dfrac{3}{36 - x^2}$

$36 - x^2 \neq 0 \implies x^2 \neq 36 \implies x \neq \pm 6$

Domain: all real numbers $x \neq \pm 6$

99. $g(t) = \ln(t - 1)$

$t - 1 > 0$

$t > 1$

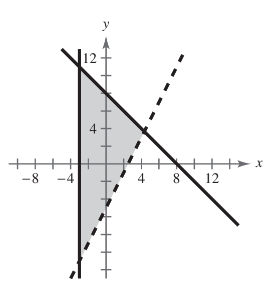
Domain: all real numbers $t > 1$

100. $f(s) = 625e^{-0.5S}$

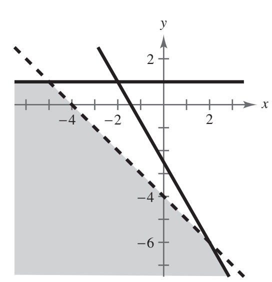
The exponential function $y = Ae^x$ is defined for all real numbers.

Domain: all real numbers

101. $\begin{cases} x + y \leq 8 \\ x \geq -3 \\ 2x - y < 5 \end{cases}$



102.



103. $[A \,\vdots\, I] = \begin{bmatrix} -4 & 1 & \vdots & 1 & 0 \\ 8 & -1 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} \\ 2R_1 + R_2 \longrightarrow \end{array} \begin{bmatrix} -4 & 1 & \vdots & 1 & 0 \\ 0 & 1 & \vdots & 2 & 1 \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \longrightarrow \\ \\ \end{array} \begin{bmatrix} -4 & 0 & \vdots & -1 & -1 \\ 0 & 1 & \vdots & 2 & 1 \end{bmatrix}$$

$$\begin{array}{r} -\frac{1}{4}R_1 \longrightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \vdots & 2 & 1 \end{bmatrix} = [I \,\vdots\, A^{-1}]$$

$$A^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ 2 & 1 \end{bmatrix}$$

104. $[A \,\vdots\, I] = \begin{bmatrix} -5 & -8 & \vdots & 1 & 0 \\ 3 & 6 & \vdots & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} R_2 \\ R_1 \end{array} \begin{bmatrix} 3 & 6 & \vdots & 0 & 1 \\ -5 & -8 & \vdots & 1 & 0 \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{3}R_1 \longrightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 2 & \vdots & 0 & \frac{1}{3} \\ -5 & -8 & \vdots & 1 & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ 5R_1 + R_2 \longrightarrow \end{array} \begin{bmatrix} 1 & 2 & \vdots & 0 & \frac{1}{3} \\ 0 & 2 & \vdots & 1 & \frac{5}{3} \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{2}R_2 \longrightarrow \end{array} \begin{bmatrix} 1 & 2 & \vdots & 0 & \frac{1}{3} \\ 0 & 1 & \vdots & \frac{1}{2} & \frac{5}{6} \end{bmatrix}$$

$$\begin{array}{r} -2R_2 + R_1 \longrightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & -1 & -\frac{4}{3} \\ 0 & 1 & \vdots & \frac{1}{2} & \frac{5}{6} \end{bmatrix} = [I \,\vdots\, A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -1 & -\frac{4}{3} \\ \frac{1}{2} & \frac{5}{6} \end{bmatrix}$$

105. $[A \,\vdots\, I] = \begin{bmatrix} -7 & 2 & 9 & \vdots & 1 & 0 & 0 \\ 2 & -4 & -6 & \vdots & 0 & 1 & 0 \\ 3 & 5 & 2 & \vdots & 0 & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} 4R_2 + R_1 \longrightarrow \\ \\ \\ \end{array} \begin{bmatrix} 1 & -14 & -15 & \vdots & 1 & 4 & 0 \\ 2 & -4 & -6 & \vdots & 0 & 1 & 0 \\ 3 & 5 & 2 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ -2R_1 + R_2 \longrightarrow \\ -3R_1 + R_3 \longrightarrow \end{array} \begin{bmatrix} 1 & -14 & -15 & \vdots & 1 & 4 & 0 \\ 0 & 24 & 24 & \vdots & -2 & -7 & 0 \\ 0 & 47 & 47 & \vdots & -3 & -12 & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -\frac{47}{24}R_2 + R_3 \longrightarrow \end{array} \begin{bmatrix} 1 & -14 & -15 & \vdots & 1 & 4 & 0 \\ 0 & 24 & 24 & \vdots & -2 & -7 & 0 \\ 0 & 0 & 0 & \vdots & \frac{11}{12} & \frac{41}{24} & 1 \end{bmatrix}$$

The zeros in Row 3 imply that the inverse does not exist.

106. $[A \,\vdots\, I] = \begin{bmatrix} -6 & 2 & 0 & \vdots & 1 & 0 & 0 \\ 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ -2 & 0 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix}$

$$\begin{array}{r} R_2 \\ R_1 \\ \\ \end{array} \begin{bmatrix} 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ -6 & 2 & 0 & \vdots & 1 & 0 & 0 \\ -2 & 0 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array} \begin{bmatrix} 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ -2 & 0 & 1 & \vdots & 0 & 0 & 1 \\ -6 & 2 & 0 & \vdots & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ 2R_1 + R_2 \longrightarrow \\ 6R_1 + R_3 \longrightarrow \end{array} \begin{bmatrix} 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ 0 & 6 & -3 & \vdots & 0 & 2 & 1 \\ 0 & 20 & -12 & \vdots & 1 & 6 & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ \frac{1}{6}R_2 \longrightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \vdots & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & 20 & -12 & \vdots & 1 & 6 & 0 \end{bmatrix}$$

—CONTINUED—

106. —CONTINUED—

$$\begin{bmatrix} 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \vdots & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & -2 & \vdots & 1 & -\frac{2}{3} & -\frac{10}{3} \end{bmatrix} \quad -20R_2 + R_3 \rightarrow \text{(row 3)}$$

$$-\tfrac{1}{2}R_3 \rightarrow \begin{bmatrix} 1 & 3 & -2 & \vdots & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \vdots & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & \vdots & -\frac{1}{2} & \frac{1}{3} & \frac{5}{3} \end{bmatrix}$$

$$-3R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} & \vdots & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \vdots & 0 & \frac{1}{3} & \frac{1}{6} \\ 0 & 0 & 1 & \vdots & -\frac{1}{2} & \frac{1}{3} & \frac{5}{3} \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{2}R_3 + R_1 \rightarrow \\ \frac{1}{2}R_3 + R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & -\frac{1}{4} & \frac{1}{6} & \frac{1}{3} \\ 0 & 1 & 0 & \vdots & -\frac{1}{4} & \frac{1}{2} & 1 \\ 0 & 0 & 1 & \vdots & -\frac{1}{2} & \frac{1}{3} & \frac{5}{3} \end{bmatrix} = [I \ \vdots \ A^{-1}]$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{6} & \frac{1}{3} \\ -\frac{1}{4} & \frac{1}{2} & 1 \\ -\frac{1}{2} & \frac{1}{3} & \frac{5}{3} \end{bmatrix}$$

Section 8.5 Applications of Matrices and Determinants

- You should be able to use Cramer's Rule to solve a system of linear equations.
- Now you should be able to solve a system of linear equations by graphing, substitution, elimination, elementary row operations on an augmented matrix, using the inverse matrix, or Cramer's Rule.
- You should be able to find the area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3).

  $$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

  The $\pm$ symbol indicates that the appropriate sign should be chosen so that the area is positive.
- You should be able to test to see if three points, (x_1, y_1), (x_2, y_2), and (x_3, y_3), are collinear.

  $$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0, \text{ if and only if they are collinear.}$$
- You should be able to find the equation of the line through (x_1, y_1) and (x_2, y_2) by evaluating.

  $$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$
- You should be able to encode and decode messages by using an invertible $n \times n$ matrix.

Vocabulary Check

1. Cramer's Rule **2.** colinear **3.** $A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ **4.** cryptogram **5.** uncoded; coded

1. $\begin{cases} 3x + 4y = -2 \\ 5x + 3y = 4 \end{cases}$

$$x = \frac{\begin{vmatrix} -2 & 4 \\ 4 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 5 & 3 \end{vmatrix}} = \frac{-22}{-11} = 2$$

$$y = \frac{\begin{vmatrix} 3 & -2 \\ 5 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 5 & 3 \end{vmatrix}} = \frac{22}{-11} = -2$$

Solution: $(2, -2)$

2. $\begin{cases} -4x - 7y = 47 \\ -x + 6y = -27 \end{cases}$

$$x = \frac{\begin{vmatrix} 47 & -7 \\ -27 & 6 \end{vmatrix}}{\begin{vmatrix} -4 & -7 \\ -1 & 6 \end{vmatrix}} = \frac{93}{-31} = -3$$

$$y = \frac{\begin{vmatrix} -4 & 47 \\ -1 & -27 \end{vmatrix}}{\begin{vmatrix} -4 & -7 \\ -1 & 6 \end{vmatrix}} = \frac{155}{-31} = -5$$

Solution: $(-3, -5)$

3. $\begin{cases} 3x + 2y = -2 \\ 6x + 4y = 4 \end{cases}$

Since $\begin{vmatrix} 3 & 2 \\ 6 & 4 \end{vmatrix} = 0$, Cramer's Rule does not apply.

The system is inconsistent in this case and has no solution.

4. $\begin{cases} 6x - 5y = 17 \\ -13x + 3y = -76 \end{cases}$

$$x = \frac{\begin{vmatrix} 17 & -5 \\ -76 & 3 \end{vmatrix}}{\begin{vmatrix} 6 & -5 \\ -13 & 3 \end{vmatrix}} = \frac{-329}{-47} = 7$$

$$y = \frac{\begin{vmatrix} 6 & 17 \\ -13 & -76 \end{vmatrix}}{\begin{vmatrix} 6 & -5 \\ -13 & 3 \end{vmatrix}} = \frac{-235}{-47} = 5$$

Solution: $(7, 5)$

5. $\begin{cases} -0.4x + 0.8y = 1.6 \\ 0.2x + 0.3y = 2.2 \end{cases}$

$$x = \frac{\begin{vmatrix} 1.6 & 0.8 \\ 2.2 & 0.3 \end{vmatrix}}{\begin{vmatrix} -0.4 & 0.8 \\ 0.2 & 0.3 \end{vmatrix}} = \frac{-1.28}{-0.28} = \frac{32}{7}$$

$$y = \frac{\begin{vmatrix} -0.4 & 1.6 \\ 0.2 & 2.2 \end{vmatrix}}{\begin{vmatrix} -0.4 & 0.8 \\ 0.2 & 0.3 \end{vmatrix}} = \frac{-1.20}{-0.28} = \frac{30}{7}$$

Solution: $\left(\frac{32}{7}, \frac{30}{7}\right)$

6. $\begin{cases} 2.4x - 1.3y = 14.63 \\ -4.6x + 0.5y = -11.51 \end{cases}$

$$x = \frac{\begin{vmatrix} 14.63 & -1.3 \\ -11.51 & 0.5 \end{vmatrix}}{\begin{vmatrix} 2.4 & -1.3 \\ -4.6 & 0.5 \end{vmatrix}} = \frac{-7.648}{-4.78} = \frac{8}{5}$$

$$y = \frac{\begin{vmatrix} 2.4 & 14.63 \\ -4.6 & -11.51 \end{vmatrix}}{\begin{vmatrix} 2.4 & -1.3 \\ -4.6 & 0.5 \end{vmatrix}} = \frac{39.674}{-4.78} = \frac{-83}{10}$$

Solution: $\left(\frac{8}{5}, -\frac{83}{10}\right)$

7. $\begin{cases} 4x - y + z = -5 \\ 2x + 2y + 3z = 10, \\ 5x - 2y + 6z = 1 \end{cases}$ $\quad D = \begin{vmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -2 & 6 \end{vmatrix} = 55$

$$x = \frac{\begin{vmatrix} -5 & -1 & 1 \\ 10 & 2 & 3 \\ 1 & -2 & 6 \end{vmatrix}}{55} = \frac{-55}{55} = -1, \quad y = \frac{\begin{vmatrix} 4 & -5 & 1 \\ 2 & 10 & 3 \\ 5 & 1 & 6 \end{vmatrix}}{55} = \frac{165}{55} = 3, \quad z = \frac{\begin{vmatrix} 4 & -1 & -5 \\ 2 & 2 & 10 \\ 5 & -2 & 1 \end{vmatrix}}{55} = \frac{110}{55} = 2$$

Solution: $(-1, 3, 2)$

8. $\begin{cases} 4x - 2y + 3z = -2 \\ 2x + 2y + 5z = 16 \\ 8x - 5y - 2z = 4 \end{cases}$

$$D = \begin{vmatrix} 4 & -2 & 3 \\ 2 & 2 & 5 \\ 8 & -5 & -2 \end{vmatrix} = -82$$

$$x = \frac{\begin{vmatrix} -2 & -2 & 3 \\ 16 & 2 & 5 \\ 4 & -5 & -2 \end{vmatrix}}{-82} = \frac{-401}{-82} = 5$$

$$y = \frac{\begin{vmatrix} 4 & -2 & 3 \\ 2 & 16 & 5 \\ 8 & 4 & -2 \end{vmatrix}}{-82} = \frac{-656}{-82} = 8$$

$$z = \frac{\begin{vmatrix} 4 & -2 & -2 \\ 2 & 2 & 16 \\ 8 & -5 & 4 \end{vmatrix}}{-82} = \frac{164}{-82} = -2$$

Solution: $(5, 8, -2)$

9. $\begin{cases} x + 2y + 3z = -3 \\ -2x + y - z = 6, \\ 3x - 3y + 2z = -11 \end{cases}$ $D = \begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & -1 \\ 3 & -3 & 2 \end{vmatrix} = 10$

$$x = \frac{\begin{vmatrix} -3 & 2 & 3 \\ 6 & 1 & -1 \\ -11 & -3 & 2 \end{vmatrix}}{10} = \frac{-20}{10} = -2$$

$$y = \frac{\begin{vmatrix} 1 & -3 & 3 \\ -2 & 6 & -1 \\ 3 & -11 & 2 \end{vmatrix}}{10} = \frac{10}{10} = 1$$

$$z = \frac{\begin{vmatrix} 1 & 2 & -3 \\ -2 & 1 & 6 \\ 3 & -3 & -11 \end{vmatrix}}{10} = \frac{-10}{10} = -1$$

Solution: $(-2, 1, -1)$

10. $\begin{cases} 5x - 4y + z = -14 \\ -x + 2y - 2z = 10 \\ 3x + y + z = 1 \end{cases}$

$$D = \begin{vmatrix} 5 & -4 & 1 \\ -1 & 2 & -2 \\ 3 & 1 & 1 \end{vmatrix} = 33$$

$$x = \frac{\begin{vmatrix} -14 & -4 & 1 \\ 10 & 2 & -2 \\ 1 & 1 & 1 \end{vmatrix}}{33} = \frac{0}{33} = 0$$

$$y = \frac{\begin{vmatrix} 5 & -14 & 1 \\ -1 & 10 & -2 \\ 3 & 1 & 1 \end{vmatrix}}{33} = \frac{99}{33} = 3$$

$$z = \frac{\begin{vmatrix} 5 & -4 & -14 \\ -1 & 2 & 10 \\ 3 & 1 & 1 \end{vmatrix}}{33} = -\frac{66}{33} = -2$$

Solution: $(0, 3, -2)$

11. $\begin{cases} 3x + 3y + 5z = 1 \\ 3x + 5y + 9z = 2, \\ 5x + 9y + 17z = 4 \end{cases}$ $D = \begin{vmatrix} 3 & 3 & 5 \\ 3 & 5 & 9 \\ 5 & 9 & 17 \end{vmatrix} = 4$

$$x = \frac{\begin{vmatrix} 1 & 3 & 5 \\ 2 & 5 & 9 \\ 4 & 9 & 17 \end{vmatrix}}{4} = 0$$

$$y = \frac{\begin{vmatrix} 3 & 1 & 5 \\ 3 & 2 & 9 \\ 5 & 4 & 17 \end{vmatrix}}{4} = -\frac{1}{2}$$

$$z = \frac{\begin{vmatrix} 3 & 3 & 1 \\ 3 & 5 & 2 \\ 5 & 9 & 4 \end{vmatrix}}{4} = \frac{1}{2}$$

Solution: $\left(0, -\frac{1}{2}, \frac{1}{2}\right)$

12. $\begin{cases} x + 2y - z = -7 \\ 2x - 2y - 2z = -8, \\ -x + 3y + 4z = 8 \end{cases}$ $D = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -2 & -2 \\ -1 & 3 & 4 \end{vmatrix} = -18$

$$x = \frac{\begin{vmatrix} -7 & 2 & -1 \\ -8 & -2 & -2 \\ 8 & 3 & 4 \end{vmatrix}}{-18} = -3, \quad y = \frac{\begin{vmatrix} 1 & -7 & -1 \\ 2 & -8 & -2 \\ -1 & 8 & 4 \end{vmatrix}}{-18} = -1, \quad z = \frac{\begin{vmatrix} 1 & 2 & -7 \\ 2 & -2 & -8 \\ -1 & 3 & 8 \end{vmatrix}}{-18} = 2$$

Solution: $(-3, -1, 2)$

13. $\begin{cases} 2x + y + 2z = 6 \\ -x + 2y - 3z = 0 \\ 3x + 2y - z = 6 \end{cases}$ $\quad D = \begin{vmatrix} 2 & 1 & 2 \\ -1 & 2 & -3 \\ 3 & 2 & -1 \end{vmatrix} = -18$

$$x = \frac{\begin{vmatrix} 6 & 1 & 2 \\ 0 & 2 & -3 \\ 6 & 2 & -1 \end{vmatrix}}{-18} = 1,\ y = \frac{\begin{vmatrix} 2 & 6 & 2 \\ -1 & 0 & -3 \\ 3 & 6 & -1 \end{vmatrix}}{-18} = 2,\ z = \frac{\begin{vmatrix} 2 & 1 & 6 \\ -1 & 2 & 0 \\ 3 & 2 & 6 \end{vmatrix}}{-18} = 1$$

Solution: $(1, 2, 1)$

14. $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 17z = 13 \end{cases}$

$$D = \begin{vmatrix} 2 & 3 & 5 \\ 3 & 5 & 9 \\ 5 & 9 & 17 \end{vmatrix} = 0$$

Cramer's Rule does not apply.

15. Vertices: $(0, 0), (3, 1), (1, 5)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 1 & 1 \\ 1 & 5 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 7 \text{ square units}$$

16. Vertices: $(0, 0), (4, 5), (5, -2)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 4 & 5 & 1 \\ 5 & -2 & 1 \end{vmatrix} = -\frac{1}{2}\begin{vmatrix} 4 & 5 \\ 5 & -2 \end{vmatrix} = \frac{33}{2} \text{ square units}$$

17. Vertices: $(-2, -3), (2, -3), (0, 4)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 2 & -3 & 1 \\ 0 & 4 & 1 \end{vmatrix} = \frac{1}{2}\left(-2\begin{vmatrix} -3 & 1 \\ 4 & 1 \end{vmatrix} - 2\begin{vmatrix} -3 & 1 \\ 4 & 1 \end{vmatrix}\right) = \frac{1}{2}(14 + 14) = 14 \text{ square units}$$

18. Vertices: $(-2, 1), (1, 6), (3, -1)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 1 & 6 & 1 \\ 3 & -1 & 1 \end{vmatrix} = -\frac{1}{2}\left(-2\begin{vmatrix} 6 & 1 \\ -1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 6 \\ 3 & -1 \end{vmatrix}\right) = -\frac{1}{2}(-14 + 2 - 19) = \frac{31}{2} \text{ square units}$$

19. Vertices: $\left(0, \frac{1}{2}\right), \left(\frac{5}{2}, 0\right), (4, 3)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & \frac{1}{2} & 1 \\ \frac{5}{2} & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix} = \frac{1}{2}\left(-\frac{1}{2}\begin{vmatrix} \frac{5}{2} & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} \frac{5}{2} & 0 \\ 4 & 3 \end{vmatrix}\right) = \frac{1}{2}\left(\frac{3}{4} + \frac{15}{2}\right) = \frac{33}{8} \text{ square units}$$

20. Vertices: $(-4, -5), (6, 10), (6, -1)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} -4 & -5 & 1 \\ 6 & 10 & 1 \\ 6 & -1 & 1 \end{vmatrix} = -\frac{1}{2}\left(6\begin{vmatrix} -5 & 1 \\ 10 & 1 \end{vmatrix} - (-1)\begin{vmatrix} -4 & 1 \\ 6 & 1 \end{vmatrix} + \begin{vmatrix} -4 & -5 \\ 6 & 10 \end{vmatrix}\right) = 55 \text{ square units}$$

21. Vertices: $(-2, 4), (2, 3), (-1, 5)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & 3 & 1 \\ -1 & 5 & 1 \end{vmatrix} = \frac{1}{2}\left[\begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix} - \begin{vmatrix} -2 & 4 \\ -1 & 5 \end{vmatrix} + \begin{vmatrix} -2 & 4 \\ 2 & 3 \end{vmatrix}\right] = \frac{1}{2}(13 + 6 - 14) = \frac{5}{2} \text{ square units}$$

22. Vertices: $(0, -2), (-1, 4), (3, 5)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} 0 & -2 & 1 \\ -1 & 4 & 1 \\ 3 & 5 & 1 \end{vmatrix} = -\frac{1}{2}\left(2\begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} -1 & 4 \\ 3 & 5 \end{vmatrix}\right) = -\frac{1}{2}(-8 - 17) = \frac{25}{2} \text{ square units}$$

23. Vertices: $(-3, 5), (2, 6), (3, -5)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} -3 & 5 & 1 \\ 2 & 6 & 1 \\ 3 & -5 & 1 \end{vmatrix} = -\frac{1}{2}\left[\begin{vmatrix} 2 & 6 \\ 3 & -5 \end{vmatrix} - \begin{vmatrix} -3 & 5 \\ 3 & -5 \end{vmatrix} + \begin{vmatrix} -3 & 5 \\ 2 & 6 \end{vmatrix}\right] = -\frac{1}{2}(-28 + 0 - 28) = 28 \text{ square units}$$

24. Vertices: $(-2, 4), (1, 5), (3, -2)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 1 & 5 & 1 \\ 3 & -2 & 1 \end{vmatrix} = -\frac{1}{2}\left(-2\begin{vmatrix} 5 & 1 \\ -2 & 1 \end{vmatrix} - \begin{vmatrix} 4 & 1 \\ -2 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & 1 \\ 5 & 1 \end{vmatrix}\right) = -\frac{1}{2}(-14 - 6 - 3) = \frac{23}{2} \text{ square units}$$

25. $$4 = \pm\frac{1}{2}\begin{vmatrix} -5 & 1 & 1 \\ 0 & 2 & 1 \\ -2 & y & 1 \end{vmatrix}$$

$$\pm 8 = -5\begin{vmatrix} 2 & 1 \\ y & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$$

$$\pm 8 = -5(2 - y) - 2(-1)$$

$$\pm 8 = 5y - 8$$

$$y = \frac{8 \pm 8}{5}$$

$$y = \frac{16}{5} \text{ or } y = 0$$

26. $$4 = \pm\frac{1}{2}\begin{vmatrix} -4 & 2 & 1 \\ -3 & 5 & 1 \\ -1 & y & 1 \end{vmatrix}$$

$$\pm 8 = \begin{vmatrix} -3 & 5 \\ -1 & y \end{vmatrix} - \begin{vmatrix} -4 & 2 \\ -1 & y \end{vmatrix} + \begin{vmatrix} -4 & 2 \\ -3 & 5 \end{vmatrix}$$

$$\pm 8 = -3y + 5 - (-4y + 2) - 20 + 6$$

$$\pm 8 = -3y + 5 + 4y - 2 - 20 + 6$$

$$\pm 8 = y - 11$$

$$y = 11 \pm 8$$

$$y = 19 \text{ or } y = 3$$

27. $$6 = \pm\frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 1 & -1 & 1 \\ -8 & y & 1 \end{vmatrix}$$

$$\pm 12 = \begin{vmatrix} 1 & -1 \\ -8 & y \end{vmatrix} - \begin{vmatrix} -2 & -3 \\ -8 & y \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ 1 & -1 \end{vmatrix}$$

$$\pm 12 = (y - 8) - (-2y - 24) + 5$$

$$\pm 12 = 3y + 21$$

$$y = \frac{-21 \pm 12}{3} = -7 \pm 4$$

$$y = -3 \text{ or } y = -11$$

28. $$6 = \pm\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 5 & -3 & 1 \\ -3 & y & 1 \end{vmatrix}$$

$$\pm 12 = \begin{vmatrix} -3 & 1 \\ y & 1 \end{vmatrix} + \begin{vmatrix} 5 & -3 \\ -3 & y \end{vmatrix}$$

$$\pm 12 = -3 - y + 5y - 9$$

$$\pm 12 = 4y - 12$$

$$y = \frac{12 \pm 12}{4} = 3 \pm 3$$

$$y = 6 \text{ or } y = 0$$

29. Vertices: $(0, 25), (10, 0), (28, 5)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 25 & 1 \\ 10 & 0 & 1 \\ 28 & 5 & 1 \end{vmatrix} = 250 \text{ square miles}$$

30. Vertices: $(0, 30), (85, 0), (20, -50)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} 0 & 30 & 1 \\ 85 & 0 & 1 \\ 20 & -50 & 1 \end{vmatrix} = 3100 \text{ square feet}$$

31. Points: $(3, -1), (0, -3), (12, 5)$

$$\begin{vmatrix} 3 & -1 & 1 \\ 0 & -3 & 1 \\ 12 & 5 & 1 \end{vmatrix} = 3\begin{vmatrix} -3 & 1 \\ 5 & 1 \end{vmatrix} + 12\begin{vmatrix} -1 & 1 \\ -3 & 1 \end{vmatrix} = 3(-8) + 12(2) = 0$$

The points are collinear.

32. Points: $(-3, -5), (6, 1), (10, 2)$

$$\begin{vmatrix} -3 & -5 & 1 \\ 6 & 1 & 1 \\ 10 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 6 & 1 \\ 10 & 2 \end{vmatrix} - \begin{vmatrix} -3 & -5 \\ 10 & 2 \end{vmatrix} + \begin{vmatrix} -3 & -5 \\ 6 & 1 \end{vmatrix} = 2 - 44 + 27 = -15 \neq 0$$

The points are not collinear.

33. Points: $\left(2, -\frac{1}{2}\right), (-4, 4), (6, -3)$

$$\begin{vmatrix} 2 & -\frac{1}{2} & 1 \\ -4 & 4 & 1 \\ 6 & -3 & 1 \end{vmatrix} = \begin{vmatrix} -4 & 4 \\ 6 & -3 \end{vmatrix} - \begin{vmatrix} 2 & -\frac{1}{2} \\ 6 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -\frac{1}{2} \\ -4 & 4 \end{vmatrix} = -12 + 3 + 6 = -3 \neq 0$$

The points are not collinear.

34. Points: $(0, 1), (4, -2), \left(-2, \frac{5}{2}\right)$

$$\begin{vmatrix} 0 & 1 & 1 \\ 4 & -2 & 1 \\ -2 & \frac{5}{2} & 1 \end{vmatrix} = -\begin{vmatrix} 4 & 1 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 4 & -2 \\ -2 & \frac{5}{2} \end{vmatrix} = -6 + 6 = 0$$

The points are collinear.

35. Points: $(0, 2), (1, 2.4), (-1, 1.6)$

$$\begin{vmatrix} 0 & 2 & 1 \\ 1 & 2.4 & 1 \\ -1 & 1.6 & 1 \end{vmatrix} = -2\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2.4 \\ -1 & 1.6 \end{vmatrix} = -2(2) + 4 = 0$$

The points are collinear.

36. Points: $(2, 3), (3, 3.5), (-1, 2)$

$$\begin{vmatrix} 2 & 3 & 1 \\ 3 & 3.5 & 1 \\ -1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 3.5 \\ -1 & 2 \end{vmatrix} - \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 3 & 3.5 \end{vmatrix} = 9.5 - 7 + (-2) = \frac{1}{2} \neq 0$$

The points are not collinear.

37.
$$\begin{vmatrix} 2 & -5 & 1 \\ 4 & y & 1 \\ 5 & -2 & 1 \end{vmatrix} = 0$$

$$2\begin{vmatrix} y & 1 \\ -2 & 1 \end{vmatrix} + 5\begin{vmatrix} 4 & 1 \\ 5 & 1 \end{vmatrix} + \begin{vmatrix} 4 & y \\ 5 & -2 \end{vmatrix} = 0$$

$$2(y + 2) + 5(-1) + (-8 - 5y) = 0$$

$$-3y - 9 = 0$$

$$y = -3$$

38.
$$\begin{vmatrix} -6 & 2 & 1 \\ -5 & y & 1 \\ -3 & 5 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -5 & y \\ -3 & 5 \end{vmatrix} - \begin{vmatrix} -6 & 2 \\ -3 & 5 \end{vmatrix} + \begin{vmatrix} -6 & 2 \\ -5 & y \end{vmatrix} = 0$$

$$-25 + 3y + 24 - 6y + 10 = 0$$

$$-3y = -9$$

$$y = 3$$

39. Points: $(0, 0), (5, 3)$

Equation: $\begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 5 & 3 & 1 \end{vmatrix} = -\begin{vmatrix} x & y \\ 5 & 3 \end{vmatrix} = 5y - 3x = 0 \implies 3x - 5y = 0$

40. Points: $(0, 0), (-2, 2)$

Equation: $\begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ -2 & 2 & 1 \end{vmatrix} = -\begin{vmatrix} x & y \\ -2 & 2 \end{vmatrix} = -(2x + 2y) = 0$ or $x + y = 0$

41. Points: $(-4, 3), (2, 1)$

Equation: $\begin{vmatrix} x & y & 1 \\ -4 & 3 & 1 \\ 2 & 1 & 1 \end{vmatrix} = x\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - y\begin{vmatrix} -4 & 1 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} -4 & 3 \\ 2 & 1 \end{vmatrix} = 2x + 6y - 10 = 0 \implies x + 3y - 5 = 0$

42. Points: $(10, 7), (-2, -7)$

Equation:

$$\begin{vmatrix} x & y & 1 \\ 10 & 7 & 1 \\ -2 & -7 & 1 \end{vmatrix} = \begin{vmatrix} 10 & 7 \\ -2 & -7 \end{vmatrix} - \begin{vmatrix} x & y \\ -2 & -7 \end{vmatrix} + \begin{vmatrix} x & y \\ 10 & 7 \end{vmatrix} = -70 + 14 - (-7x + 2y) + 7x - 10y = 0 \text{ or } 7x - 6y - 28 = 0$$

43. Points: $\left(-\frac{1}{2}, 3\right), \left(\frac{5}{2}, 1\right)$

Equation: $\begin{vmatrix} x & y & 1 \\ -\frac{1}{2} & 3 & 1 \\ \frac{5}{2} & 1 & 1 \end{vmatrix} = x\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - y\begin{vmatrix} -\frac{1}{2} & 1 \\ \frac{5}{2} & 1 \end{vmatrix} + \begin{vmatrix} -\frac{1}{2} & 3 \\ \frac{5}{2} & 1 \end{vmatrix} = 2x + 3y - 8 = 0$

44. Points: $\left(\frac{2}{3}, 4\right), (6, 12)$

Equation: $\begin{vmatrix} x & y & 1 \\ \frac{2}{3} & 4 & 1 \\ 6 & 12 & 1 \end{vmatrix} = \begin{vmatrix} \frac{2}{3} & 4 \\ 6 & 12 \end{vmatrix} - \begin{vmatrix} x & y \\ 6 & 12 \end{vmatrix} + \begin{vmatrix} x & y \\ \frac{2}{3} & 4 \end{vmatrix} = -16 - (12x - 6y) + 4x - \frac{2}{3}y = 0$ or $3x - 2y + 6 = 0$

45. The uncoded row matrices are the rows of the 7×3 matrix on the left.

$$\begin{array}{ccc} \text{T} & \text{R} & \text{O} \\ \text{U} & \text{B} & \text{L} \\ \text{E} & & \text{I} \\ \text{N} & & \text{R} \\ \text{I} & \text{V} & \text{E} \\ \text{R} & & \text{C} \\ \text{I} & \text{T} & \text{Y} \end{array} \begin{bmatrix} 20 & 18 & 15 \\ 21 & 2 & 12 \\ 5 & 0 & 9 \\ 14 & 0 & 18 \\ 9 & 22 & 5 \\ 18 & 0 & 3 \\ 9 & 20 & 25 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -52 & 10 & 27 \\ -49 & 3 & 34 \\ -49 & 13 & 27 \\ -94 & 22 & 54 \\ 1 & 1 & -7 \\ 0 & -12 & 9 \\ -121 & 41 & 55 \end{bmatrix}$$

Solution: -52 10 27 -49 3 34 -49 13 27 -94 22 54 1 1 -7 0 -12 9 -121 41 55

46. $[16 \quad 12 \quad 5]\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix} = [43 \quad 6 \quad 9]$

$[1 \quad 19 \quad 5]\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix} = [-38 \quad -45 \quad -13]$

$[0 \quad 19 \quad 5]\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 3 & 1 \end{bmatrix} = [-42 \quad -47 \quad -14]$

$[14 \quad 4 \quad 0]\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 1 & 1 \end{bmatrix} = [44 \quad 16 \quad 10]$

$[13 \quad 15 \quad 14]\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix} = [49 \quad 9 \quad 12]$

$[5 \quad 25 \quad 0]\begin{bmatrix} 4 & 2 & 1 \\ -3 & -3 & -1 \\ 3 & 2 & 1 \end{bmatrix} = [-55 \quad -65 \quad -20]$

Solution: Uncoded 1×3 matrices: $[16 \quad 12 \quad 5], [1 \quad 19 \quad 5], [0 \quad 19 \quad 5], [14 \quad 4 \quad 0], [13 \quad 15 \quad 14], [5 \quad 25 \quad 0]$
Encoded 1×3 matrices: $[43 \quad 6 \quad 9], [-38 \quad -45 \quad -13], [-42 \quad -47 \quad -14]$,
$[44 \quad 16 \quad 10], [49 \quad 9 \quad 12], [-55 \quad -65 \quad -20]$

Encoded message: 43 6 9 −38 −45 −13 −42 −47 −14
44 16 10 49 9 12 −55 −65 −20

In Exercises 47–50, use the matrix $A = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{2} \\ \mathbf{3} & \mathbf{7} & \mathbf{9} \\ \mathbf{-1} & \mathbf{-4} & \mathbf{-7} \end{bmatrix}$.

47. C A L L __ A T __ N O O N
[3 1 12] [12 0 1] [20 0 14] [15 15 14]

$[3 \quad 1 \quad 12]A = [-6 \quad -35 \quad -69]$

$[12 \quad 0 \quad 1]A = [11 \quad 20 \quad 17]$

$[20 \quad 0 \quad 14]A = [6 \quad -16 \quad -58]$

$[15 \quad 15 \quad 14]A = [46 \quad 79 \quad 67]$

Cryptogram: −6 −35 −69 11 20 17 6 −16 −58 46 79 67

48. I C E B E R G __ D E A D __ A H E A D
[9 3 5] [2 5 18] [7 0 4] [5 1 4] [0 1 8] [5 1 4]

$[9 \quad 3 \quad 5]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [13 \quad 19 \quad 10]$

$[2 \quad 5 \quad 18]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [-1 \quad -33 \quad -77]$

$[7 \quad 0 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [3 \quad -2 \quad -14]$

—CONTINUED—

48. —CONTINUED—

$$[5 \quad 1 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [4 \quad 1 \quad -9]$$

$$[0 \quad 1 \quad 8]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [-5 \quad -25 \quad -47]$$

$$[5 \quad 1 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [4 \quad 1 \quad -9]$$

Cryptogram: 13 19 10 -1 -33 -77 3 -2 -14
4 1 -9 -5 -25 -47 4 1 -9

49. H A P P Y _ B I R T H D A Y _
[8 1 16] [16 25 0] [2 9 18] [20 8 4] [1 25 0]

$[\,8 \quad 1 \quad 16]\, A = [-5 \quad -41 \quad -87]$

$[16 \quad 25 \quad 0]\, A = [91 \quad 207 \quad 257]$

$[\,2 \quad 9 \quad 18]\, A = [11 \quad -5 \quad -41]$

$[20 \quad 8 \quad 4]\, A = [40 \quad 80 \quad 84]$

$[\,1 \quad 25 \quad 0]\, A = [76 \quad 177 \quad 227]$

Cryptogram: -5 -41 -87 91 207 257 11 -5 -41 40 80 84 76 177 227

50. O P E R A T I O N _ O V E R L O A D
[15 16 5] [18 1 20] [9 15 14] [0 15 22] [5 18 12] [15 1 4]

$$[15 \quad 16 \quad 5]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [58 \quad 122 \quad 139]$$

$$[18 \quad 1 \quad 20]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [1 \quad -37 \quad -95]$$

$$[9 \quad 15 \quad 14]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [40 \quad 67 \quad 55]$$

$$[0 \quad 15 \quad 22]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [23 \quad 17 \quad -19]$$

$$[5 \quad 18 \quad 12]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [47 \quad 88 \quad 88]$$

$$[15 \quad 1 \quad 4]\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix} = [14 \quad 21 \quad 11]$$

Cryptogram: 58 122 139 1 -37 -95 40 67 55 23 17 -19 47 88 88 14 21 11

51. $A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

$$\begin{bmatrix} 11 & 21 \\ 64 & 112 \\ 25 & 50 \\ 29 & 53 \\ 23 & 46 \\ 40 & 75 \\ 55 & 92 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 16 & 16 \\ 25 & 0 \\ 14 & 5 \\ 23 & 0 \\ 25 & 5 \\ 1 & 18 \end{bmatrix} \begin{matrix} \text{H} & \text{A} \\ \text{P} & \text{P} \\ \text{Y} & \_\_ \\ \text{N} & \text{E} \\ \text{W} & \_\_ \\ \text{Y} & \text{E} \\ \text{A} & \text{R} \end{matrix}$$

Message: HAPPY NEW YEAR

52. $A^{-1} = \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix}$

$[-136 \quad 58]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [2 \quad 18]$ B R

$[-173 \quad 72]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [15 \quad 14]$ O N

$[-120 \quad 51]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [3 \quad 15]$ C O

$[-95 \quad 38]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [19 \quad 0]$ S __

$[-178 \quad 73]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [23 \quad 9]$ W I

$[-70 \quad 28]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [14 \quad 0]$ N __

$[-242 \quad 101]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [19 \quad 21]$ S U

$[-115 \quad 47]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [16 \quad 5]$ P E

$[-90 \quad 36]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [18 \quad 0]$ R __

$[-115 \quad 49]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [2 \quad 15]$ B O

$[-199 \quad 82]\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix} = [23 \quad 12]$ W L

Message: BRONCOS WIN SUPER BOWL

53. $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix}$

$$\begin{bmatrix} 9 & -1 & -9 \\ 38 & -19 & -19 \\ 28 & -9 & -19 \\ -80 & 25 & 41 \\ -64 & 21 & 31 \\ 9 & -5 & -4 \end{bmatrix} \begin{bmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 12 & 1 \\ 19 & 19 & 0 \\ 9 & 19 & 0 \\ 3 & 1 & 14 \\ 3 & 5 & 12 \\ 5 & 4 & 0 \end{bmatrix} \begin{matrix} \text{C} & \text{L} & \text{A} \\ \text{S} & \text{S} & \_\_ \\ \text{I} & \text{S} & \_\_ \\ \text{C} & \text{A} & \text{N} \\ \text{C} & \text{E} & \text{L} \\ \text{E} & \text{D} & \_\_ \end{matrix}$$

Message: CLASS IS CANCELED

54. $A^{-1} = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix}$

$[112 \quad -140 \quad 83]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [8 \quad 1 \quad 22]$ H A V

$[19 \quad -25 \quad 13]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [5 \quad 0 \quad 1]$ E _ A

$[72 \quad -76 \quad 61]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [0 \quad 7 \quad 18]$ _ G R

Message: HAVE A GREAT WEEKEND

$[95 \quad -118 \quad 71]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [5 \quad 1 \quad 20]$ E A T

$[20 \quad 21 \quad 38]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [0 \quad 23 \quad 5]$ _ W E

$[35 \quad -23 \quad 36]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [5 \quad 11 \quad 5]$ E K E

$[42 \quad -48 \quad 32]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [14 \quad 4 \quad 0]$ N D _

55. $A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 20 & 17 & -15 \\ -12 & -56 & -104 \\ 1 & -25 & -65 \\ 62 & 143 & 181 \end{bmatrix}\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = \begin{array}{c|ccc} \begin{bmatrix} 19 & 5 & 14 \\ 4 & 0 & 16 \\ 12 & 1 & 14 \\ 5 & 19 & 0 \end{bmatrix} & \begin{matrix} \text{S} & \text{E} & \text{N} \\ \text{D} & \_ & \text{P} \\ \text{L} & \text{A} & \text{N} \\ \text{E} & \text{S} & \_ \end{matrix} \end{array}$$

Message: SEND PLANES

56. $[13 \quad -9 \quad -59]\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = [18 \quad 5 \quad 20]$ R E T

$[61 \quad 112 \quad 106]\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = [21 \quad 18 \quad 14]$ U R N

$[-17 \quad -73 \quad -131]\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = [0 \quad 1 \quad 20]$ _ A T

Message: RETURN AT DAWN

$[11 \quad 24 \quad 29]\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = [0 \quad 4 \quad 1]$ _ D A

$[65 \quad 144 \quad 172]\begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} = [23 \quad 14 \quad 0]$ W N _

57. Let A be the 2×2 matrix needed to decode the message.

$$\begin{bmatrix} -18 & -18 \\ 1 & 16 \end{bmatrix} A = \begin{bmatrix} 0 & 18 \\ 15 & 14 \end{bmatrix} \begin{matrix} & \text{R} \\ \text{O} & \text{N} \end{matrix}$$

$$A = \begin{bmatrix} -18 & -18 \\ 1 & 16 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 18 \\ 15 & 14 \end{bmatrix} = \begin{bmatrix} -\frac{8}{135} & -\frac{1}{15} \\ \frac{1}{270} & \frac{1}{15} \end{bmatrix} \begin{bmatrix} 0 & 18 \\ 15 & 14 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 21 \\ -15 & -10 \\ -13 & -13 \\ 5 & 10 \\ 5 & 25 \\ 5 & 19 \\ -1 & 6 \\ 20 & 40 \\ -18 & -18 \\ 1 & 16 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 5 \\ 5 & 20 \\ 0 & 13 \\ 5 & 0 \\ 20 & 15 \\ 14 & 9 \\ 7 & 8 \\ 20 & 0 \\ 0 & 18 \\ 15 & 14 \end{bmatrix} \begin{matrix} \text{M} & \text{E} \\ \text{E} & \text{T} \\ \_ & \text{M} \\ \text{E} & \_ \\ \text{T} & \text{O} \\ \text{N} & \text{I} \\ \text{G} & \text{H} \\ \text{T} & \_ \\ \_ & \text{R} \\ \text{O} & \text{N} \end{matrix}$$

Message: MEET ME TONIGHT RON

58. (a) $n = 3$; $\sum_{i=1}^{n} x_i = 0 + 1 + 2 = 3$; $\sum_{i=1}^{n} x_i^2 = 0^2 + 1^2 + 2^2 = 5$; $\sum_{i=1}^{n} x_i^3 = 0^3 + 1^3 + 2^3 = 9$;

$\sum_{i=1}^{n} x_i^4 = 0^4 + 1^4 + 2^4 = 17$; $\sum_{i=1}^{n} y_i = 8965 + 9176 + 9406 = 27{,}547$

$$\sum_{i=1}^{n} x_i y_i = 0(8965) + 1(9176) + 2(9406) = 27{,}988$$

$$\sum_{i=1}^{n} x_i^2 y_i = 0^2(8965) + 1^2(9176) + 2^2(9406) = 46{,}800$$

System: $\begin{cases} 3c + 3b + 5a = 27{,}547 \\ 3c + 5b + 9a = 27{,}988 \\ 5c + 9b + 17a = 46{,}800 \end{cases}$

(b) $D = \begin{vmatrix} 3 & 3 & 5 \\ 3 & 5 & 9 \\ 5 & 9 & 17 \end{vmatrix} = 4$

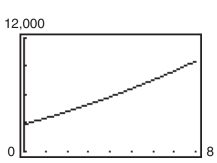
$$c = \frac{\begin{vmatrix} 27{,}547 & 3 & 5 \\ 27{,}988 & 5 & 9 \\ 46{,}800 & 9 & 17 \end{vmatrix}}{4} = \frac{35{,}860}{4} = 8965$$

$$b = \frac{\begin{vmatrix} 3 & 27{,}547 & 5 \\ 3 & 27{,}988 & 9 \\ 5 & 46{,}800 & 17 \end{vmatrix}}{4} = \frac{806}{4} = 201.5$$

$$a = \frac{\begin{vmatrix} 3 & 3 & 27{,}547 \\ 3 & 5 & 27{,}988 \\ 5 & 9 & 46{,}800 \end{vmatrix}}{4} = \frac{38}{4} = 9.5$$

The least squares regression parabola is $y = 9.5t^2 + 201.5t + 8965$.

(c)



(d) The intersection of the regression parabola and the line $y = 10{,}000$ is about $t = 4.3$, so the number of cases waiting to be tried will reach 10,000 in about 2004.

59. False. In Cramer's Rule, the **denominator** is the determinant of the coefficient matrix.

60. True. If the determinant of the coefficient matrix is zero, the solution of the system would result in division by zero which is undefined.

61. False. If the determinant of the coefficient matrix is zero, the system has either no solution or infinitely many solutions.

62. Answers will vary. To solve a system of linear equations you can use graphing, substitution, elimination, elementary row operations on an augmented matrix (Gaussian elimination with back–substitution or Gauss-Jordan elimination), the inverse of a matrix, or Cramer's Rule.

63. $\begin{cases} -x - 7y = -22 & \text{Equation 1} \\ 5x + y = -26 & \text{Equation 2} \end{cases}$

$$\begin{aligned} -5x - 35y &= -110 && (5)\text{Eq.1} \\ 5x + y &= -26 \\ -34y &= -136 && \text{Add equations.} \\ y &= 4 \\ -x - 7(4) &= -22 \\ x &= -6 \end{aligned}$$

Solution: $(-6, 4)$

64. $\begin{cases} 3x + 8y = 11 & \text{Equation 1} \\ -2x + 12y = -16 & \text{Equation 2} \end{cases}$

$\begin{cases} 9x + 24y = 33 & (3)\text{Eq.1} \\ 4x - 24y = 32 & (-2)\text{Eq.2} \end{cases}$

$$\begin{aligned} 13x &= 65 && \text{Add equations.} \\ x &= \tfrac{65}{13} = 5 \end{aligned}$$

$3(5) + 8y = 11 \implies 8y = -4 \implies y = -\frac{1}{2}$

Solution: $\left(5, -\frac{1}{2}\right)$

65. $\begin{cases} -x - 3y + 5z = -14 \\ 4x + 2y - z = -1 \\ 5x - 3y + 2z = -11 \end{cases}$

$$A^{-1} = \begin{bmatrix} -1 & -3 & 5 \\ 4 & 2 & -1 \\ 5 & -3 & 2 \end{bmatrix}^{-1} = \frac{1}{72}\begin{bmatrix} -1 & 9 & 7 \\ 13 & 27 & -19 \\ 22 & 18 & -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} -14 \\ -1 \\ -11 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$$

Solution: $(-1, 0, -3)$

66. $\begin{cases} 5x - y - z = 7 \\ -2x + 3y + z = -5 \\ 4x + 10y - 5z = -37 \end{cases}$

$$A^{-1} = \begin{bmatrix} 5 & -1 & -1 \\ -2 & 3 & 1 \\ 4 & 10 & -5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{25}{87} & \frac{5}{29} & -\frac{2}{87} \\ \frac{2}{29} & \frac{7}{29} & \frac{1}{29} \\ \frac{32}{87} & \frac{18}{29} & -\frac{13}{87} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 7 \\ -5 \\ -37 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix}$$

Solution: $(2, -2, 5)$

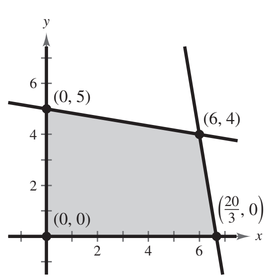
67. Objective function: $z = 6x + 4y$

Constraints: $x \ge 0$

$y \ge 0$

$x + 6y \le 30$

$6x + y \le 40$



At $(0, 0)$: $z = 6(0) + 4(0) = 0$

At $(0, 5)$: $z = 6(0) + 4(5) = 20$

At $(6, 4)$: $z = 6(6) + 4(4) = 52$

At $\left(\frac{20}{3}, 0\right)$: $z = 6\left(\frac{20}{3}\right) + 4(0) = 40$

The minimum value of 0 occurs at $(0, 0)$.

The maximum value of 52 occurs at $(6, 4)$.

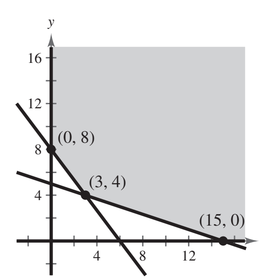
68. Objective function: $z = 6x + 7y$

Constraints: $x \ge 0$

$y \ge 0$

$4x + 3y \ge 24$

$x + 3y \ge 15$



Since the region is unbounded, there is no maximum value of the objective function. To find the minimum value, check the vertices.

At $(0, 8)$: $z = 6(0) + 7(8) = 56$

At $(3, 4)$: $z = 6(3) + 7(4) = 46$

At $(15, 0)$: $z = 6(15) + 7(0) = 90$

The minimum value of 46 occurs at $(3, 4)$.

Review Exercises for Chapter 8

1. $\begin{bmatrix} -4 \\ 0 \\ 5 \end{bmatrix}$

Order: 3×1

2. $\begin{bmatrix} 3 & -1 & 0 & 6 \\ -2 & 7 & 1 & 4 \end{bmatrix}$

Since the matrix has two rows and four columns, its order is 2×4.

3. $[3]$

Order: 1×1

4. $[6 \quad 2 \quad -5 \quad 8 \quad 0]$

Since the matrix has one row and five columns, its order is 1×5.

5. $\begin{cases} 3x - 10y = 15 \\ 5x + 4y = 22 \end{cases}$

$$\left[\begin{array}{cc|c} 3 & -10 & 15 \\ 5 & 4 & 22 \end{array}\right]$$

6. $\begin{cases} 8x - 7y + 4z = 12 \\ 3x - 5y + 2z = 20 \\ 5x + 3y - 3z = 26 \end{cases}$

$$\left[\begin{array}{ccc|c} 8 & -7 & 4 & 12 \\ 3 & -5 & 2 & 20 \\ 5 & 3 & -3 & 26 \end{array}\right]$$

7. $\left[\begin{array}{ccc|c} 5 & 1 & 7 & -9 \\ 4 & 2 & 0 & 10 \\ 9 & 4 & 2 & 3 \end{array}\right]$

$$\begin{cases} 5x + y + 7z = -9 \\ 4x + 2y = 10 \\ 9x + 4y + 2z = 3 \end{cases}$$

8. $\left[\begin{array}{cccc|c} 13 & 16 & 7 & 3 & 2 \\ 1 & 21 & 8 & 5 & 12 \\ 4 & 10 & -4 & 3 & -1 \end{array}\right]$

$$\begin{cases} 13x + 16y + 7z + 3w = 2 \\ x + 21y + 8z + 5w = 12 \\ 4x + 10y - 4z + 3w = -1 \end{cases}$$

9. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix}$

$$\begin{array}{r} R_1 \\ R_2 \\ \\ \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -2R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -2 & -4 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ 2R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -\frac{1}{2}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

10. $\begin{bmatrix} 4 & 8 & 16 \\ 3 & -1 & 2 \\ -2 & 10 & 12 \end{bmatrix}$

$$\begin{array}{r} \frac{1}{4}R_1 \rightarrow \\ \\ -\frac{1}{2}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & 2 \\ 1 & -5 & -6 \end{bmatrix}$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -10 \\ 0 & -7 & -10 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -10 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{r} \\ -\frac{1}{7}R_2 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & \frac{10}{7} \\ 0 & 0 & 0 \end{bmatrix}$$

11. $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{array} \begin{cases} x + 2y + 3z = 9 \\ y - 2z = 2 \\ z = 0 \end{cases}$

$$y - 2(0) = 2 \Rightarrow y = 2$$

$$x + 2(2) + 3(0) = 9 \Rightarrow x = 5$$

Solution: $(5, 2, 0)$

12. $\begin{cases} x + 3y - 9z = 4 \\ y - z = 10 \\ z = -2 \end{cases}$

$$y - (-2) = 10$$

$$y = 8$$

$$x + 3(8) - 9(-2) = 4$$

$$x = -38$$

Solution: $(-38, 8, -2)$

13. $\begin{bmatrix} 1 & -5 & 4 & \vdots & 1 \\ 0 & 1 & 2 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & 4 \end{bmatrix} \begin{matrix} \Rightarrow \\ \Rightarrow \\ \Rightarrow \end{matrix} \begin{cases} x - 5y + 4z = 1 \\ y + 2z = 3 \\ z = 4 \end{cases}$

$y + 2(4) = 3 \Rightarrow y = -5$

$x - 5(-5) + 4(4) = 1 \Rightarrow x = -40$

Solution: $(-40, -5, 4)$

14. $\begin{cases} x - 8y = -2 \\ y - z = -7 \\ z = 1 \end{cases}$

$y - 1 = -7$

$y = -6$

$x - 8(-6) = -2$

$x = -50$

Solution: $(-50, -6, 1)$

15. $\begin{bmatrix} 5 & 4 & \vdots & 2 \\ -1 & 1 & \vdots & -22 \end{bmatrix}$

$4R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 8 & \vdots & -86 \\ -1 & 1 & \vdots & -22 \end{bmatrix}$

$R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 8 & \vdots & -86 \\ 0 & 9 & \vdots & -108 \end{bmatrix}$

$\frac{1}{9}R_2 \rightarrow \begin{bmatrix} 1 & 8 & \vdots & -86 \\ 0 & 1 & \vdots & -12 \end{bmatrix}$

$\begin{cases} x + 8y = -86 \\ y = -12 \end{cases}$

$y = -12$

$x + 8(-12) = -86 \Rightarrow x = 10$

Solution: $(10, -12)$

16. $\begin{bmatrix} 2 & -5 & \vdots & 2 \\ 3 & -7 & \vdots & 1 \end{bmatrix}$

$\frac{1}{2}R_1 \rightarrow \begin{bmatrix} 1 & -\frac{5}{2} & \vdots & 1 \\ 3 & -7 & \vdots & 1 \end{bmatrix}$

$-3R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -\frac{5}{2} & \vdots & 1 \\ 0 & \frac{1}{2} & \vdots & -2 \end{bmatrix}$

$2R_2 \rightarrow \begin{bmatrix} 1 & -\frac{5}{2} & \vdots & 1 \\ 0 & 1 & \vdots & -4 \end{bmatrix}$

$\begin{cases} x - \frac{5}{2}y = 1 \\ y = -4 \end{cases}$

$y = -4$

$x - \frac{5}{2}(-4) = 1 \Rightarrow x = -9$

Solution: $(-9, -4)$

17. $\begin{bmatrix} 0.3 & -0.1 & \vdots & -0.13 \\ 0.2 & -0.3 & \vdots & -0.25 \end{bmatrix}$

$\begin{matrix} 10R_1 \rightarrow \\ 10R_2 \rightarrow \end{matrix} \begin{bmatrix} 3 & -1 & \vdots & -1.3 \\ 2 & -3 & \vdots & -2.5 \end{bmatrix}$

$-R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 1.2 \\ 2 & -3 & \vdots & -2.5 \end{bmatrix}$

$-2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 1.2 \\ 0 & -7 & \vdots & -4.9 \end{bmatrix}$

$-\frac{1}{7}R_2 \rightarrow \begin{bmatrix} 1 & 2 & \vdots & 1.2 \\ 0 & 1 & \vdots & 0.7 \end{bmatrix}$

$\begin{cases} x + 2y = 1.2 \\ y = 0.7 \end{cases}$

$y = 0.7$

$x + 2(0.7) = 1.2 \Rightarrow x = -0.2$

Solution: $(-0.2, 0.7) = \left(-\frac{1}{5}, \frac{7}{10}\right)$

18. $\begin{bmatrix} 0.2 & -0.1 & \vdots & 0.07 \\ 0.4 & -0.5 & \vdots & -0.01 \end{bmatrix}$

$\begin{matrix} 5R_1 \rightarrow \\ -2R_1 + R_2 \rightarrow \end{matrix} \begin{bmatrix} 1 & -0.5 & \vdots & 0.35 \\ 0 & -0.3 & \vdots & -0.15 \end{bmatrix}$

$-\frac{1}{0.3}R_2 \rightarrow \begin{bmatrix} 1 & -0.5 & \vdots & 0.35 \\ 0 & 1 & \vdots & 0.5 \end{bmatrix}$

$\begin{cases} x - 0.5y = 0.35 \\ y = 0.5 \end{cases}$

$y = 0.5$

$x - 0.5(0.5) = 0.35 \Rightarrow x = 0.6$

Solution: $(0.6, 0.5) = \left(\frac{3}{5}, \frac{1}{2}\right)$

19.
$$\begin{bmatrix} 2 & 3 & 1 & \vdots & 10 \\ 2 & -3 & -3 & \vdots & 22 \\ 4 & -2 & 3 & \vdots & -2 \end{bmatrix}$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{array}\begin{bmatrix} 2 & 3 & 1 & \vdots & 10 \\ 0 & -6 & -4 & \vdots & 12 \\ 0 & -8 & 1 & \vdots & -22 \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{2}R_1 \rightarrow \\ -\frac{1}{6}R_2 \rightarrow \\ \\ \end{array}\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & \vdots & 5 \\ 0 & 1 & \frac{2}{3} & \vdots & -2 \\ 0 & -8 & 1 & \vdots & -22 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ 8R_2 + R_3 \rightarrow \end{array}\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & \vdots & 5 \\ 0 & 1 & \frac{2}{3} & \vdots & -2 \\ 0 & 0 & \frac{19}{3} & \vdots & -38 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ \frac{3}{19}R_3 \rightarrow \end{array}\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} & \vdots & 5 \\ 0 & 1 & \frac{2}{3} & \vdots & -2 \\ 0 & 0 & 1 & \vdots & -6 \end{bmatrix}$$

$z = -6$

$y + \frac{2}{3}(-6) = -2 \implies y = 2$

$x + \frac{3}{2}(2) + \frac{1}{2}(-6) = 5 \implies x = 5$

Solution: $(5, 2, -6)$

20.
$$\begin{bmatrix} 2 & 3 & 3 & \vdots & 3 \\ 6 & 6 & 12 & \vdots & 13 \\ 12 & 9 & -1 & \vdots & 2 \end{bmatrix}$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -6R_1 + R_3 \rightarrow \end{array}\begin{bmatrix} 2 & 3 & 3 & \vdots & 3 \\ 0 & -3 & 3 & \vdots & 4 \\ 0 & -9 & -19 & \vdots & -16 \end{bmatrix}$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -3R_2 + R_3 \rightarrow \end{array}\begin{bmatrix} 2 & 0 & 6 & \vdots & 7 \\ 0 & -3 & 3 & \vdots & 4 \\ 0 & 0 & -28 & \vdots & -28 \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{2}R_1 \rightarrow \\ -\frac{1}{3}R_2 \rightarrow \\ -\frac{1}{28}R_3 \rightarrow \end{array}\begin{bmatrix} 1 & 0 & 3 & \vdots & \frac{7}{2} \\ 0 & 1 & -1 & \vdots & -\frac{4}{3} \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

$$\begin{cases} x \quad + 3z = \frac{7}{2} \\ y - \quad z = -\frac{4}{3} \\ \qquad\quad z = 1 \end{cases}$$

$z = 1$

$y - 1 = -\frac{4}{3} \implies y = -\frac{1}{3}$

$x + 3(1) = \frac{7}{2} \implies x = \frac{1}{2}$

Solution: $\left(\frac{1}{2}, -\frac{1}{3}, 1\right)$

21.
$$\begin{bmatrix} 2 & 1 & 2 & \vdots & 4 \\ 2 & 2 & 0 & \vdots & 5 \\ 2 & -1 & 6 & \vdots & 2 \end{bmatrix}$$

$$\begin{array}{r} \\ -R_1 + R_2 \rightarrow \\ -R_1 + R_3 \rightarrow \end{array}\begin{bmatrix} 2 & 1 & 2 & \vdots & 4 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & -2 & 4 & \vdots & -2 \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array}\begin{bmatrix} 2 & 0 & 4 & \vdots & 3 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\begin{array}{r} \frac{1}{2}R_1 \rightarrow \\ \\ \\ \end{array}\begin{bmatrix} 1 & 0 & 2 & \vdots & \frac{3}{2} \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

Let $z = a$, then:

$y - 2a = 1 \implies y = 2a + 1$

$x + 2a = \frac{3}{2} \implies x = -2a + \frac{3}{2}$

Solution: $\left(-2a + \frac{3}{2}, 2a + 1, a\right)$ where a is any real number

22.
$$\begin{bmatrix} 1 & 2 & 6 & \vdots & 1 \\ 2 & 5 & 15 & \vdots & 4 \\ 3 & 1 & 3 & \vdots & -6 \end{bmatrix}$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{array}\begin{bmatrix} 1 & 2 & 6 & \vdots & 1 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & -5 & -15 & \vdots & -9 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ 5R_2 + R_3 \rightarrow \end{array}\begin{bmatrix} 1 & 2 & 6 & \vdots & 1 \\ 0 & 1 & 3 & \vdots & 2 \\ 0 & 0 & 0 & \vdots & 1 \end{bmatrix}$$

Because the last row consists of all zeros except for the last entry, the system is inconsistent and there is no solution.

23.

$$\left[\begin{array}{rrrr|r} 2 & 1 & 1 & 0 & 6 \\ 0 & -2 & 3 & -1 & 9 \\ 3 & 3 & -2 & -2 & -11 \\ 1 & 0 & 1 & 3 & 14 \end{array}\right]$$

$$\begin{array}{r} -R_4 + R_1 \\ \\ \\ \\ \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & -2 & 3 & -1 & 9 \\ 3 & 3 & -2 & -2 & -11 \\ 1 & 0 & 1 & 3 & 14 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -3R_1 + R_3 \rightarrow \\ -R_1 + R_4 \rightarrow \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & -2 & 3 & -1 & 9 \\ 0 & 0 & -2 & 7 & 13 \\ 0 & -1 & 1 & 6 & 22 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_4 + R_2 \rightarrow \\ \\ \\ \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & 1 & 0 & -19 & -57 \\ 0 & 0 & -2 & 7 & 13 \\ 0 & -1 & 1 & 6 & 22 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \\ R_2 + R_4 \rightarrow \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & 1 & 0 & -19 & -57 \\ 0 & 0 & -2 & 7 & 13 \\ 0 & 0 & 1 & -13 & -35 \end{array}\right]$$

$$\begin{array}{r} \\ \\ R_4 \\ R_3 \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & 1 & 0 & -19 & -57 \\ 0 & 0 & 1 & -13 & -35 \\ 0 & 0 & -2 & 7 & 13 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \\ 2R_3 + R_4 \rightarrow \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & 1 & 0 & -19 & -57 \\ 0 & 0 & 1 & -13 & -35 \\ 0 & 0 & 0 & -19 & -57 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \\ \frac{1}{19}R_4 \rightarrow \end{array}\left[\begin{array}{rrrr|r} 1 & 1 & 0 & -3 & -8 \\ 0 & 1 & 0 & -19 & -57 \\ 0 & 0 & 1 & -13 & -35 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$w = 3$

$z - 13(3) = -35 \implies z = 4$

$y - 19(3) = -57 \implies y = 0$

$x + 0 - 3(3) = -8 \implies x = 1$

Solution: $(1, 0, 4, 3)$

24.

$$\left[\begin{array}{rrrr|r} 1 & 2 & 0 & 1 & 3 \\ 0 & -3 & 3 & 0 & 0 \\ 4 & 4 & 1 & 2 & 0 \\ 2 & 0 & 1 & 0 & 3 \end{array}\right]$$

$$\begin{array}{r} \\ -\frac{1}{3}R_2 \rightarrow \\ -4R_1 + R_3 \rightarrow \\ -2R_1 + R_4 \rightarrow \end{array}\left[\begin{array}{rrrr|r} 1 & 2 & 0 & 1 & 3 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -4 & 1 & -2 & -12 \\ 0 & -4 & 1 & -2 & -3 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \\ -R_3 + R_4 \rightarrow \end{array}\left[\begin{array}{rrrr|r} 1 & 2 & 0 & 1 & 3 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -4 & 1 & -2 & -12 \\ 0 & 0 & 0 & 0 & 9 \end{array}\right]$$

Because the last row consists of all zeros except for the last entry, the system is inconsistent and there is no solution.

25.

$$\left[\begin{array}{rrr|r} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right]$$

$$\begin{array}{r} -R_1 \rightarrow \\ \\ \\ \end{array}\left[\begin{array}{rrr|r} 1 & -1 & -2 & -1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right]$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -5R_1 + R_3 \rightarrow \end{array}\left[\begin{array}{rrr|r} 1 & -1 & -2 & -1 \\ 0 & 5 & 5 & 0 \\ 0 & 9 & 12 & 9 \end{array}\right]$$

$$\begin{array}{r} \\ \frac{1}{5}R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrr|r} 1 & -1 & -2 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 9 & 12 & 9 \end{array}\right]$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ -9R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrr|r} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 9 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{3}R_3 \rightarrow \end{array}\left[\begin{array}{rrr|r} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\begin{array}{r} R_3 + R_1 \rightarrow \\ -R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 2, y = -3, z = 3$

Solution: $(2, -3, 3)$

26. $\begin{cases}4x + 4y + 4z = 5\\ 4x - 2y - 8z = 1\\ 5x + 3y + 8z = 6\end{cases}$

$$\left[\begin{array}{rrr|r}4 & 4 & 4 & 5\\ 4 & -2 & -8 & 1\\ 5 & 3 & 8 & 6\end{array}\right]$$

$$\begin{array}{r}\frac{1}{4}R_1 \to\\ \\ \\ \end{array}\left[\begin{array}{rrr|r}1 & 1 & 1 & \frac{5}{4}\\ 4 & -2 & -8 & 1\\ 5 & 3 & 8 & 6\end{array}\right]$$

$$\begin{array}{r}\\ -4R_1 + R_2 \to\\ -5R_1 + R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & 1 & 1 & \frac{5}{4}\\ 0 & -6 & -12 & -4\\ 0 & -2 & 3 & -\frac{1}{4}\end{array}\right]$$

$$\begin{array}{r}\\ -\frac{1}{6}R_2 \to\\ \\ \end{array}\left[\begin{array}{rrr|r}1 & 1 & 1 & \frac{5}{4}\\ 0 & 1 & 2 & \frac{2}{3}\\ 0 & -2 & 3 & -\frac{1}{4}\end{array}\right]$$

$$\begin{array}{r}-R_2 + R_1 \to\\ \\ 2R_2 + R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & 0 & -1 & \frac{7}{12}\\ 0 & 1 & 2 & \frac{2}{3}\\ 0 & 0 & 7 & \frac{13}{12}\end{array}\right]$$

$$\begin{array}{r}\\ \\ \frac{1}{7}R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & 0 & -1 & \frac{7}{12}\\ 0 & 1 & 2 & \frac{2}{3}\\ 0 & 0 & 1 & \frac{13}{84}\end{array}\right]$$

$$\begin{array}{r}R_3 + R_1 \to\\ -2R_3 + R_2 \to\\ \\ \end{array}\left[\begin{array}{rrr|r}1 & 0 & 0 & \frac{31}{42}\\ 0 & 1 & 0 & \frac{5}{14}\\ 0 & 0 & 1 & \frac{13}{84}\end{array}\right]$$

$x = \frac{31}{42}$

$y = \frac{5}{14}$

$z = \frac{13}{84}$

Solution: $\left(\frac{31}{42}, \frac{5}{14}, \frac{13}{84}\right)$

27.
$$\left[\begin{array}{rrr|r}2 & -1 & 9 & -8\\ -1 & -3 & 4 & -15\\ 5 & 2 & -1 & 17\end{array}\right]$$

$$\begin{array}{r}R_2 + R_1 \to\\ \\ \\ \end{array}\left[\begin{array}{rrr|r}1 & -4 & 13 & -23\\ -1 & -3 & 4 & -15\\ 5 & 2 & -1 & 17\end{array}\right]$$

$$\begin{array}{r}\\ R_1 + R_2 \to\\ -5R_1 + R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & -4 & 13 & -23\\ 0 & -7 & 17 & -38\\ 0 & 22 & -66 & 132\end{array}\right]$$

$$\begin{array}{r}\\ R_3\\ R_2\end{array}\left[\begin{array}{rrr|r}1 & -4 & 13 & -23\\ 0 & 22 & -66 & 132\\ 0 & -7 & 17 & -38\end{array}\right]$$

$$\begin{array}{r}\\ \frac{1}{22}R_2 \to\\ \\ \end{array}\left[\begin{array}{rrr|r}1 & -4 & 13 & -23\\ 0 & 1 & -3 & 6\\ 0 & -7 & 17 & -38\end{array}\right]$$

$$\begin{array}{r}\\ \\ 7R_2 + R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & -4 & 13 & -23\\ 0 & 1 & -3 & 6\\ 0 & 0 & -4 & 4\end{array}\right]$$

$$\begin{array}{r}\\ \\ -\frac{1}{4}R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & -4 & 13 & -23\\ 0 & 1 & -3 & 6\\ 0 & 0 & 1 & -1\end{array}\right]$$

$$\begin{array}{r}4R_2 + R_1 \to\\ \\ \\ \end{array}\left[\begin{array}{rrr|r}1 & 0 & 1 & 1\\ 0 & 1 & -3 & 6\\ 0 & 0 & 1 & -1\end{array}\right]$$

$$\begin{array}{r}-R_3 + R_1 \to\\ 3R_3 + R_2 \to\\ \\ \end{array}\left[\begin{array}{rrr|r}1 & 0 & 0 & 2\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & -1\end{array}\right]$$

$x = 2, y = 3, z = -1$

Solution: $(2, 3, -1)$

28. $\begin{cases}-3x + y + 7z = -20\\ 5x - 2y - z = 34\\ -x + y + 4z = -8\end{cases}$

$$\left[\begin{array}{rrr|r}-3 & 1 & 7 & -20\\ 5 & -2 & -1 & 34\\ -1 & 1 & 4 & -8\end{array}\right]$$

$$\begin{array}{r}R_3\\ \\ R_1\end{array}\left[\begin{array}{rrr|r}-1 & 1 & 4 & -8\\ 5 & -2 & -1 & 34\\ -3 & 1 & 7 & -20\end{array}\right]$$

$$\begin{array}{r}-1R_1 \to\\ \\ \\ \end{array}\left[\begin{array}{rrr|r}1 & -1 & -4 & 8\\ 5 & -2 & -1 & 34\\ -3 & 1 & 7 & -20\end{array}\right]$$

$$\begin{array}{r}\\ -5R_1 + R_2 \to\\ 3R_1 + R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & -1 & -4 & 8\\ 0 & 3 & 19 & -6\\ 0 & -2 & -5 & 4\end{array}\right]$$

$$\begin{array}{r}\\ \frac{1}{3}R_2 \to\\ \\ \end{array}\left[\begin{array}{rrr|r}1 & -1 & -4 & 8\\ 0 & 1 & \frac{19}{3} & -2\\ 0 & -2 & -5 & 4\end{array}\right]$$

$$\begin{array}{r}R_2 + R_1 \to\\ \\ 2R_2 + R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & 0 & \frac{7}{3} & 6\\ 0 & 1 & \frac{19}{3} & -2\\ 0 & 0 & \frac{23}{3} & 0\end{array}\right]$$

$$\begin{array}{r}\\ \\ \frac{3}{23}R_3 \to\end{array}\left[\begin{array}{rrr|r}1 & 0 & \frac{7}{3} & 6\\ 0 & 1 & \frac{19}{3} & -2\\ 0 & 0 & 1 & 0\end{array}\right]$$

$$\begin{array}{r}-\frac{7}{3}R_3 + R_1 \to\\ \\ -\frac{19}{3}R_3 + R_2 \to\end{array}\left[\begin{array}{rrr|r}1 & 0 & 0 & 6\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & 0\end{array}\right]$$

$x = 6, y = -2, z = 0$

Solution: $(6, -2, 0)$

29. Use the reduced row-echelon form feature of a graphing utility.

$$\left[\begin{array}{rrrr:r} 3 & -1 & 5 & -2 & -44 \\ 1 & 6 & 4 & -1 & 1 \\ 5 & -1 & 1 & 3 & -15 \\ 0 & 4 & -1 & -8 & 58 \end{array}\right] \Rightarrow \left[\begin{array}{rrrr:r} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & -10 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$x = 2, y = 6, z = -10, w = -3$

Solution: $(2, 6, -10, -3)$

30. Use the reduced row-echelon form feature of the graphing utility.

$$\left[\begin{array}{rrr:r} 4 & 12 & 2 & 20 \\ 1 & 6 & 4 & 12 \\ 1 & 6 & 1 & 8 \\ -2 & -10 & -2 & -10 \end{array}\right] \Rightarrow \left[\begin{array}{rrr:r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

The system is inconsistent and there is no solution.

31. $\begin{bmatrix} -1 & x \\ y & 9 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ -7 & 9 \end{bmatrix} \Rightarrow x = 12$ and $y = -7$

32. $\begin{bmatrix} -1 & 0 \\ x & 5 \\ -4 & y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 8 & 5 \\ -4 & 0 \end{bmatrix} \Rightarrow x = 8, y = 0$

33. $\begin{bmatrix} x+3 & -4 & 4y \\ 0 & -3 & 2 \\ -2 & y+5 & 6x \end{bmatrix} = \begin{bmatrix} 5x-1 & -4 & 44 \\ 0 & -3 & 2 \\ -2 & 16 & 6 \end{bmatrix}$

$$\left.\begin{aligned} x + 3 &= 5x - 1 \\ 4y &= 44 \\ y + 5 &= 16 \\ 6x &= 6 \end{aligned}\right\} x = 1 \text{ and } y = 11$$

34. $\begin{bmatrix} -9 & 4 & 2 & -5 \\ 0 & -3 & 7 & -4 \\ 6 & -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -9 & 4 & x-10 & -5 \\ 0 & -3 & 7 & 2y \\ \frac{1}{2}x & -1 & 1 & 0 \end{bmatrix}$

$$\left.\begin{aligned} 6 &= \tfrac{1}{2}x \\ 2 &= x - 10 \\ -4 &= 2y \end{aligned}\right\} x = 12, y = -2$$

35. (a) $A + B = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 15 & 13 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 5 & -12 \\ -9 & -3 \end{bmatrix}$

(c) $4A = 4\begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ 12 & 20 \end{bmatrix}$

(d) $A + 3B = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} + 3\begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -9 & 30 \\ 36 & 24 \end{bmatrix} = \begin{bmatrix} -7 & 28 \\ 39 & 29 \end{bmatrix}$

36. (a) $A + B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 20 & 40 \\ 15 & 30 \end{bmatrix} = \begin{bmatrix} 5+4 & 4+12 \\ -7+20 & 2+40 \\ 11+15 & 2+30 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 13 & 42 \\ 26 & 32 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 12 \\ 20 & 40 \\ 15 & 30 \end{bmatrix} = \begin{bmatrix} 5-4 & 4-12 \\ -7-20 & 2-40 \\ 11-15 & 2-30 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ -27 & -38 \\ -4 & -28 \end{bmatrix}$

(c) $4A = 4\begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 16 \\ -28 & 8 \\ 44 & 8 \end{bmatrix}$

(d) $A + 3B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + 3\begin{bmatrix} 4 & 12 \\ 20 & 40 \\ 15 & 30 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + \begin{bmatrix} 12 & 36 \\ 60 & 120 \\ 45 & 90 \end{bmatrix} = \begin{bmatrix} 17 & 40 \\ 53 & 122 \\ 56 & 92 \end{bmatrix}$

37. (a) $A + B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ -3 & 14 \\ 31 & 42 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -11 & -10 \\ -9 & -38 \end{bmatrix}$

(c) $4A = 4\begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 16 \\ -28 & 8 \\ 44 & 8 \end{bmatrix}$

(d) $A + 3B = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + 3\begin{bmatrix} 0 & 3 \\ 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 9 \\ 12 & 36 \\ 60 & 120 \end{bmatrix} = \begin{bmatrix} 5 & 13 \\ 5 & 38 \\ 71 & 122 \end{bmatrix}$

38. (a) $A + B$ is not possible. A and B do not have the same order.

(b) $A - B$ is not possible. A and B do not have the same order.

(c) $4A = 4[6 \quad -5 \quad 7] = [24 \quad -20 \quad 28]$

(d) $A + 3B$ is not possible. A and B do not have the same order.

39. $\begin{bmatrix} 7 & 3 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 10 & -20 \\ 14 & -3 \end{bmatrix} = \begin{bmatrix} 7+10 & 3-20 \\ -1+14 & 5-3 \end{bmatrix} = \begin{bmatrix} 17 & -17 \\ 13 & 2 \end{bmatrix}$

40. Since the matrices are not of the same order, the operation cannot be performed.

41. $-2\begin{bmatrix} 1 & 2 \\ 5 & -4 \\ 6 & 0 \end{bmatrix} + 8\begin{bmatrix} 7 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ -10 & 8 \\ -12 & 0 \end{bmatrix} + \begin{bmatrix} 56 & 8 \\ 8 & 16 \\ 8 & 32 \end{bmatrix} = \begin{bmatrix} 54 & 4 \\ -2 & 24 \\ -4 & 32 \end{bmatrix}$

42. $$-\begin{bmatrix} 8 & -1 & 8 \\ -2 & 4 & 12 \\ 0 & -6 & 0 \end{bmatrix} - 5\begin{bmatrix} -2 & 0 & -4 \\ 3 & -1 & 1 \\ 6 & 12 & -8 \end{bmatrix} = \begin{bmatrix} -8 & 1 & -8 \\ 2 & -4 & -12 \\ 0 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 20 \\ -15 & 5 & -5 \\ -30 & -60 & 40 \end{bmatrix}$$
$$= \begin{bmatrix} -8+10 & 1+0 & -8+20 \\ 2-15 & -4+5 & -12-5 \\ 0-30 & 6-60 & 0+40 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 12 \\ -13 & 1 & -17 \\ -30 & -54 & 40 \end{bmatrix}$$

43. $3\begin{bmatrix} 8 & -2 & 5 \\ 1 & 3 & -1 \end{bmatrix} + 6\begin{bmatrix} 4 & -2 & -3 \\ 2 & 7 & 6 \end{bmatrix} = \begin{bmatrix} 24 & -6 & 15 \\ 3 & 9 & -3 \end{bmatrix} + \begin{bmatrix} 24 & -12 & -18 \\ 12 & 42 & 36 \end{bmatrix} = \begin{bmatrix} 48 & -18 & -3 \\ 15 & 51 & 33 \end{bmatrix}$

44. $-5\begin{bmatrix} 2 & 0 \\ 7 & -2 \\ 8 & 2 \end{bmatrix} + 4\begin{bmatrix} 4 & -2 \\ 6 & 11 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -11 & 54 \\ -44 & 2 \end{bmatrix}$

45. $$X = 3A - 2B = 3\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} -14 & -4 \\ 7 & -17 \\ -17 & -2 \end{bmatrix}$$

46. $X = \frac{1}{6}(4A + 3B) = \frac{1}{6}\left(4\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}\right) = \frac{1}{6}\left(\begin{bmatrix} -16 & 0 \\ 4 & -20 \\ -12 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -6 & 3 \\ 12 & 12 \end{bmatrix}\right) = \frac{1}{6}\begin{bmatrix} -16 + 3 & 0 + 6 \\ 4 - 6 & -20 + 3 \\ -12 + 12 & 8 + 12 \end{bmatrix}$

$$= \frac{1}{6}\begin{bmatrix} -13 & 6 \\ -2 & -17 \\ 0 & 20 \end{bmatrix} = \begin{bmatrix} -\frac{13}{6} & 1 \\ -\frac{1}{3} & -\frac{17}{6} \\ 0 & \frac{10}{3} \end{bmatrix}$$

47. $X = \frac{1}{3}[B - 2A] = \frac{1}{3}\left(\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} - 2\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix}\right)$

$$= \frac{1}{3}\begin{bmatrix} 9 & 2 \\ -4 & 11 \\ 10 & 0 \end{bmatrix} = \begin{bmatrix} 3 & \frac{2}{3} \\ -\frac{4}{3} & \frac{11}{3} \\ \frac{10}{3} & 0 \end{bmatrix}$$

48. $X = \frac{1}{3}(2A - 5B) = \frac{1}{3}\left(2\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}\right) = \frac{1}{3}\left(\begin{bmatrix} -8 & 0 \\ 2 & -10 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} -5 & -10 \\ 10 & -5 \\ -20 & -20 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} -8 - 5 & 0 - 10 \\ 2 + 10 & -10 - 5 \\ -6 - 20 & 4 - 20 \end{bmatrix}$

$$= \frac{1}{3}\begin{bmatrix} -13 & -10 \\ 12 & -15 \\ -26 & -16 \end{bmatrix} = \begin{bmatrix} -\frac{13}{3} & -\frac{10}{3} \\ 4 & -5 \\ -\frac{26}{3} & -\frac{16}{3} \end{bmatrix}$$

49. A and B are both 2×2 so AB exists.

$$AB = \begin{bmatrix} 2 & -2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} -3 & 10 \\ 12 & 8 \end{bmatrix} = \begin{bmatrix} 2(-3) + (-2)(12) & 2(10) + (-2)(8) \\ 3(-3) + 5(12) & 3(10) + 5(8) \end{bmatrix} = \begin{bmatrix} -30 & 4 \\ 51 & 70 \end{bmatrix}$$

50. Not possible because the number of columns of A does not equal the number of rows of B.

51. Since A is 3×2 and B is 2×2, AB exists.

$$AB = \begin{bmatrix} 5 & 4 \\ -7 & 2 \\ 11 & 2 \end{bmatrix}\begin{bmatrix} 4 & 12 \\ 20 & 40 \end{bmatrix} = \begin{bmatrix} 5(4) + 4(20) & 5(12) + 4(40) \\ -7(4) + 2(20) & -7(12) + 2(40) \\ 11(4) + 2(20) & 11(12) + 2(40) \end{bmatrix} = \begin{bmatrix} 100 & 220 \\ 12 & -4 \\ 84 & 212 \end{bmatrix}$$

52. $AB = \begin{bmatrix} 6 & -5 & 7 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ 8 \end{bmatrix} = [6(-1) - 5(4) + 7(8)] = [30]$

53. $\begin{bmatrix} 1 & 2 \\ 5 & -4 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1(6) + 2(4) & 1(-2) + 2(0) & 1(8) + 2(0) \\ 5(6) + (-4)(4) & 5(-2) + (-4)(0) & 5(8) + (-4)(0) \\ 6(6) + (0)(4) & 6(-2) + (0)(0) & 6(8) + (0)(0) \end{bmatrix}$

$$= \begin{bmatrix} 14 & -2 & 8 \\ 14 & -10 & 40 \\ 36 & -12 & 48 \end{bmatrix}$$

54. Not possible because the number of columns of the first matrix does not equal the number of rows of the second matrix.

55. $\begin{bmatrix} 1 & 5 & 6 \\ 2 & -4 & 0 \end{bmatrix}\begin{bmatrix} 6 & 4 \\ -2 & 0 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} 1(6)+5(-2)+6(8) & 1(4)+5(0)+6(0) \\ 2(6)-4(-2)+0(8) & 2(4)-4(0)+0(0) \end{bmatrix}$

$= \begin{bmatrix} 44 & 4 \\ 20 & 8 \end{bmatrix}$

56. $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 4 & -3 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1(4) & 1(-3)+3(3) & 1(2)+3(-1)+2(2) \\ 0 & 2(3) & 2(-1)+(-4)(2) \\ 0 & 0 & 3(2) \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 & 3 \\ 0 & 6 & -10 \\ 0 & 0 & 6 \end{bmatrix}$

57. $\begin{bmatrix} 4 \\ 6 \end{bmatrix}\begin{bmatrix} 6 & -2 \end{bmatrix} = \begin{bmatrix} 4(6) & 4(-2) \\ 6(6) & 6(-2) \end{bmatrix} = \begin{bmatrix} 24 & -8 \\ 36 & -12 \end{bmatrix}$

58. $\begin{bmatrix} 4 & -2 & 6 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4(-2) -2(0)+6(2) & 4(1)-2(-3)+6(0) \end{bmatrix}$

$= \begin{bmatrix} 4 & 10 \end{bmatrix}$

59. $\begin{bmatrix} 2 & 1 \\ 6 & 0 \end{bmatrix}\left(\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 0 & 4 \end{bmatrix}\right) = \begin{bmatrix} 2 & 1 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ -3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2(2)+1(-3) & 2(6)+1(5) \\ 6(2)+0 & 6(6)+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 17 \\ 12 & 36 \end{bmatrix}$

60. $-3\begin{bmatrix} 1 & -1 \\ 4 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 5 & -3 \end{bmatrix}\right) = \begin{bmatrix} -3 & 3 \\ -12 & -6 \end{bmatrix}\begin{bmatrix} 0(1)+3(5) & 0(0)+3(-3) \\ 1(1)+2(5) & 1(0)+2(-3) \end{bmatrix}$

$= \begin{bmatrix} -3 & 3 \\ -12 & -6 \end{bmatrix}\begin{bmatrix} 15 & -9 \\ 11 & -6 \end{bmatrix}$

$= \begin{bmatrix} -3(15)+3(11) & -3(-9)+3(-6) \\ -12(15)-6(11) & -12(-9)-6(-6) \end{bmatrix}$

$= \begin{bmatrix} -12 & 9 \\ -246 & 144 \end{bmatrix}$

61. $\begin{bmatrix} 4 & 1 \\ 11 & -7 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 & 6 \\ 2 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 14 & -22 & 22 \\ 19 & -41 & 80 \\ 42 & -66 & 66 \end{bmatrix}$

62. $\begin{bmatrix} -2 & 3 & 10 \\ 4 & -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -5 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 13 & 24 \\ 20 & 4 \end{bmatrix}$

63. $0.95A = 0.95\begin{bmatrix} 80 & 120 & 140 \\ 40 & 100 & 80 \end{bmatrix} = \begin{bmatrix} 76 & 114 & 133 \\ 38 & 95 & 76 \end{bmatrix}$

64. $1.2A = 1.2\begin{bmatrix} 80 & 70 & 90 & 40 \\ 50 & 30 & 80 & 20 \\ 90 & 60 & 100 & 50 \end{bmatrix} = \begin{bmatrix} 96 & 84 & 108 & 48 \\ 60 & 36 & 96 & 24 \\ 108 & 72 & 120 & 60 \end{bmatrix}$

65. $BA = \begin{bmatrix} 10.25 & 14.50 & 17.75 \end{bmatrix} \begin{bmatrix} 8200 & 7400 \\ 6500 & 9800 \\ 5400 & 4800 \end{bmatrix} = \begin{bmatrix} \$274{,}150 & \$303{,}150 \end{bmatrix}$

The merchandise shipped to warehouse 1 is worth \$274,150, and the merchandise shipped to warehouse 2 is worth \$303,150.

66. (a) $T = \begin{bmatrix} 120 & 80 & 20 \end{bmatrix}$

(b) $TC = \begin{bmatrix} 120 & 80 & 20 \end{bmatrix} \begin{bmatrix} 0.07 & 0.095 \\ 0.10 & 0.08 \\ 0.28 & 0.25 \end{bmatrix} = \begin{bmatrix} 22 & 22.8 \end{bmatrix}$

Your cost with company A is \$22.00. Your cost with company B is \$22.80.

67. $AB = \begin{bmatrix} -4 & -1 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} -4(-2) + (-1)(7) & -4(-1) + (-1)(4) \\ 7(-2) + 2(7) & 7(-1) + 2(4) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$BA = \begin{bmatrix} -2 & -1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} -4 & -1 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} -2(-4) + (-1)(7) & -2(-1) + (-1)(2) \\ 7(-4) + 4(7) & 7(-1) + 4(2) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

68. $AB = \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$BA = \begin{bmatrix} -2 & 1 \\ -11 & 5 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 11 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

69. $AB = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} -2 & -3 & 1 \\ 3 & 3 & -1 \\ 2 & 4 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1(-2) + 1(3) + 0(2) & 1(-3) + 1(3) + 0(4) & 1(1) + 1(-1) + 0(-1) \\ 1(-2) + 0(3) + 1(2) & 1(-3) + 0(3) + 1(4) & 1(1) + 0(-1) + 1(-1) \\ 6(-2) + 2(3) + 3(2) & 6(-3) + 2(3) + 3(4) & 6(1) + 2(-1) + 3(-1) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$BA = \begin{bmatrix} -2 & -3 & 1 \\ 3 & 3 & -1 \\ 2 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 6 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -2(1) + (-3)(1) + 1(6) & -2(1) + (-3)(0) + 1(2) & -2(0) + (-3)(1) + 1(3) \\ 3(1) + 3(1) + (-1)(6) & 3(1) + 3(0) + (-1)(2) & 3(0) + 3(1) + (-1)(3) \\ 2(1) + 4(1) + (-1)(6) & 2(1) + 4(0) + (-1)(2) & 2(0) + 4(1) + (-1)(3) \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

70. $AB = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 8 & -4 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 & \frac{1}{2} \\ -3 & 1 & \frac{1}{2} \\ 2 & -2 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$BA = \begin{bmatrix} -2 & 1 & \frac{1}{2} \\ -3 & 1 & \frac{1}{2} \\ 2 & -2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 8 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

71. $[A \,\vdots\, I] = \left[\begin{array}{cc:cc} -6 & 5 & 1 & 0 \\ -5 & 4 & 0 & 1 \end{array}\right]$

$-\frac{1}{6}R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & -\frac{5}{6} & -\frac{1}{6} & 0 \\ -5 & 4 & 0 & 1 \end{array}\right]$

$5R_1 + R_2 \rightarrow \left[\begin{array}{cc:cc} 1 & -\frac{5}{6} & -\frac{1}{6} & 0 \\ 0 & -\frac{1}{6} & -\frac{5}{6} & 1 \end{array}\right]$

$-6R_2 \rightarrow \left[\begin{array}{cc:cc} 1 & -\frac{5}{6} & -\frac{1}{6} & 0 \\ 0 & 1 & 5 & -6 \end{array}\right]$

$\frac{5}{6}R_2 + R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & 0 & 4 & -5 \\ 0 & 1 & 5 & -6 \end{array}\right] = [I \,\vdots\, A^{-1}]$

$A^{-1} = \begin{bmatrix} 4 & -5 \\ 5 & -6 \end{bmatrix}$

72. $[A \,\vdots\, I] = \left[\begin{array}{cc:cc} -3 & -5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right]$

$2R_2 + R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & 1 & 1 & 2 \\ 2 & 3 & 0 & 1 \end{array}\right]$

$-2R_1 + R_2 \rightarrow \left[\begin{array}{cc:cc} 1 & 1 & 1 & 2 \\ 0 & 1 & -2 & -3 \end{array}\right]$

$-R_2 + R_1 \rightarrow \left[\begin{array}{cc:cc} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & -3 \end{array}\right] = [I \,\vdots\, A^{-1}]$

$A^{-1} = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

73. $[A \,\vdots\, I] = \left[\begin{array}{ccc:ccc} -1 & -2 & -2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ 1 & 4 & 7 & 0 & 0 & 1 \end{array}\right]$

$-R_1 \rightarrow \left[\begin{array}{ccc:ccc} 1 & 2 & 2 & -1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ 1 & 4 & 7 & 0 & 0 & 1 \end{array}\right]$

$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -R_1 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 2 & 2 & -1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 & 0 \\ 0 & 2 & 5 & 1 & 0 & 1 \end{array}\right]$

$\begin{array}{r} -2R_2 + R_1 \rightarrow \\ \\ -2R_2 + R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & -4 & -7 & -2 & 0 \\ 0 & 1 & 3 & 3 & 1 & 0 \\ 0 & 0 & -1 & -5 & -2 & 1 \end{array}\right]$

$\begin{array}{r} -4R_3 + R_1 \rightarrow \\ 3R_3 + R_2 \rightarrow \\ -R_3 \rightarrow \end{array} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 13 & 6 & -4 \\ 0 & 1 & 0 & -12 & -5 & 3 \\ 0 & 0 & 1 & 5 & 2 & -1 \end{array}\right] = [I \,\vdots\, A^{-1}]$

$A^{-1} = \begin{bmatrix} 13 & 6 & -4 \\ -12 & -5 & 3 \\ 5 & 2 & -1 \end{bmatrix}$

74. $$[A \;\vdots\; I] = \left[\begin{array}{rrrcrrr} 0 & -2 & 1 & \vdots & 1 & 0 & 0 \\ -5 & -2 & -3 & \vdots & 0 & 1 & 0 \\ 7 & 3 & 4 & \vdots & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} R_3 \\ \\ R_1 \end{array}\left[\begin{array}{rrrcrrr} 7 & 3 & 4 & \vdots & 0 & 0 & 1 \\ -5 & -2 & -3 & \vdots & 0 & 1 & 0 \\ 0 & -2 & 1 & \vdots & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ \\ \\ \end{array}\left[\begin{array}{rrrcrrr} 2 & 1 & 1 & \vdots & 0 & 1 & 1 \\ -5 & -2 & -3 & \vdots & 0 & 1 & 0 \\ 0 & -2 & 1 & \vdots & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ 5R_1 + 2R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcrrr} 2 & 1 & 1 & \vdots & 0 & 1 & 1 \\ 0 & 1 & -1 & \vdots & 0 & 7 & 5 \\ 0 & -2 & 1 & \vdots & 1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ 2R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{rrrcrrr} 2 & 0 & 2 & \vdots & 0 & -6 & -4 \\ 0 & 1 & -1 & \vdots & 0 & 7 & 5 \\ 0 & 0 & -1 & \vdots & 1 & 14 & 10 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{2}R_1 \rightarrow \\ \\ -R_3 \rightarrow \end{array}\left[\begin{array}{rrrcrrr} 1 & 0 & 1 & \vdots & 0 & -3 & -2 \\ 0 & 1 & -1 & \vdots & 0 & 7 & 5 \\ 0 & 0 & 1 & \vdots & -1 & -14 & -10 \end{array}\right]$$

$$\begin{array}{r} -R_3 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ \\ \end{array}\left[\begin{array}{rrrcrrr} 1 & 0 & 0 & \vdots & 1 & 11 & 8 \\ 0 & 1 & 0 & \vdots & -1 & -7 & -5 \\ 0 & 0 & 1 & \vdots & -1 & -14 & -10 \end{array}\right] = [I \;\vdots\; A^{-1}]$$

$$A^{-1} = \begin{bmatrix} 1 & 11 & 8 \\ -1 & -7 & -5 \\ -1 & -14 & -10 \end{bmatrix}$$

75. $$\begin{bmatrix} 2 & 0 & 3 \\ -1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{2}{3} & -\frac{5}{6} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

76. $$A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & -3 & 1 \\ -1 & 18 & 16 \end{bmatrix}$$

A^{-1} does not exist.

77. $$\begin{bmatrix} 1 & 3 & 1 & 6 \\ 4 & 4 & 2 & 6 \\ 3 & 4 & 1 & 2 \\ -1 & 2 & -1 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} -3 & 6 & -\frac{11}{2} & \frac{7}{2} \\ 1 & -2 & 2 & -1 \\ 7 & -15 & \frac{29}{2} & -\frac{19}{2} \\ -1 & \frac{5}{2} & -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 6 & -5.5 & 3.5 \\ 1 & -2 & 2 & -1 \\ 7 & -15 & 14.5 & -9.5 \\ -1 & 2.5 & -2.5 & 1.5 \end{bmatrix}$$

78. $$A = \begin{bmatrix} 8 & 0 & 2 & 8 \\ 4 & -2 & 0 & -2 \\ 1 & 2 & 1 & 4 \\ -1 & 4 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2.5 & 3 & 7 & -2 \\ -4 & 4.5 & 11 & -3 \\ 14.5 & -16 & -40 & 12 \\ -1 & 1 & 3 & -1 \end{bmatrix}$$

79. $$A = \begin{bmatrix} -7 & 2 \\ -8 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-7(2) - 2(-8)}\begin{bmatrix} 2 & -2 \\ 8 & -7 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ 8 & -7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 4 & -\frac{7}{2} \end{bmatrix}$$

80. $$A = \begin{bmatrix} 10 & 4 \\ 7 & 3 \end{bmatrix}$$

$ad - bc = (10)(3) - (4)(7) = 2$

$$A^{-1} = \frac{1}{10(3) - 4(7)}\begin{bmatrix} 3 & -4 \\ -7 & 10 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ -7 & 10 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -2 \\ -\frac{7}{2} & 5 \end{bmatrix}$$

81. $A = \begin{bmatrix} -\frac{1}{2} & 20 \\ \frac{3}{10} & -6 \end{bmatrix}$

$$A^{-1} = \frac{1}{-\frac{1}{2}(-6) - 20\left(\frac{3}{10}\right)}\begin{bmatrix} -6 & -20 \\ -\frac{3}{10} & -\frac{1}{2} \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -6 & -20 \\ -\frac{3}{10} & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & \frac{20}{3} \\ \frac{1}{10} & \frac{1}{6} \end{bmatrix}$$

82. $A = \begin{bmatrix} -\frac{3}{4} & \frac{5}{2} \\ -\frac{4}{5} & -\frac{8}{3} \end{bmatrix}$

$$ad - bc = \left(-\frac{3}{4}\right)\left(-\frac{8}{3}\right) - \left(\frac{5}{2}\right)\left(-\frac{4}{5}\right) = 2 + 2 = 4$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -\frac{8}{3} & -\frac{5}{2} \\ \frac{4}{5} & -\frac{3}{4} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -\frac{5}{8} \\ \frac{1}{5} & -\frac{3}{16} \end{bmatrix}$$

83. $\begin{cases} -x + 4y = 8 \\ 2x - 7y = -5 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ -5 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 7(8) + 4(-5) \\ 2(8) + 1(-5) \end{bmatrix} = \begin{bmatrix} 36 \\ 11 \end{bmatrix}$$

Solution: $(36, 11)$

84. $\begin{cases} 5x - y = 13 \\ -9x + 2y = -24 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -9 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -24 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 9 & 5 \end{bmatrix}\begin{bmatrix} 13 \\ -24 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Solution: $(2, -3)$

85. $\begin{cases} -3x + 10y = 8 \\ 5x - 17y = -13 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & 10 \\ 5 & -17 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ -13 \end{bmatrix} = \begin{bmatrix} -17 & -10 \\ -5 & -3 \end{bmatrix}\begin{bmatrix} 8 \\ -13 \end{bmatrix}$$

$$= \begin{bmatrix} -17(8) + (-10)(-13) \\ -5(8) + (-3)(-13) \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \end{bmatrix}$$

Solution: $(-6, -1)$

86. $\begin{cases} 4x - 2y = -10 \\ -19x + 9y = 47 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -19 & 9 \end{bmatrix}^{-1}\begin{bmatrix} -10 \\ 47 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{9}{2} & -1 \\ -\frac{19}{2} & -2 \end{bmatrix}\begin{bmatrix} -10 \\ 47 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Solution: $(-2, 1)$

87. $\begin{cases} 3x + 2y - z = 6 \\ x - y + 2z = -1 \\ 5x + y + z = 7 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & 2 & -1 \\ 1 & -1 & 2 \\ 5 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ -1 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ 3 & \frac{8}{3} & -\frac{7}{3} \\ 2 & \frac{7}{3} & -\frac{5}{3} \end{bmatrix}\begin{bmatrix} 6 \\ -1 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -1(6) - 1(-1) + 1(7) \\ 3(6) + \frac{8}{3}(-1) - \frac{7}{3}(7) \\ 2(6) + \frac{7}{3}(-1) - \frac{5}{3}(7) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

Solution: $(2, -1, -2)$

88. $\begin{cases} -x + 4y - 2z = 12 \\ 2x - 9y + 5z = -25 \\ -x + 5y - 4z = 10 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 4 & -2 \\ 2 & -9 & 5 \\ -1 & 5 & -4 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ -25 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -6 & -2 \\ -3 & -2 & -1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 12 \\ -25 \\ 10 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}$$

Solution: $(-2, 4, 3)$

89. $\begin{cases} -2x + y + 2z = -13 \\ -x - 4y + z = -11 \\ -y - z = 0 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 \\ -1 & -4 & 1 \\ 0 & -1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} -13 \\ -11 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{5}{9} & \frac{1}{9} & -1 \\ \frac{1}{9} & -\frac{2}{9} & 0 \\ -\frac{1}{9} & \frac{2}{9} & -1 \end{bmatrix}\begin{bmatrix} -13 \\ -11 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{5}{9}(-13) + \frac{1}{9}(-11) - 1(0) \\ \frac{1}{9}(-13) - \frac{2}{9}(-11) + 0(0) \\ -\frac{1}{9}(-13) + \frac{2}{9}(-11) - 1(0) \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix}$$

Solution: $(6, 1, -1)$

90. $\begin{cases} 3x - y + 5z = -14 \\ -x + y + 6z = 8 \\ -8x + 4y - z = 44 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -1 & 5 \\ -1 & 1 & 6 \\ -8 & 4 & -1 \end{bmatrix}^{-1} \begin{bmatrix} -14 \\ 8 \\ 44 \end{bmatrix} = \begin{bmatrix} \frac{25}{6} & -\frac{19}{6} & \frac{11}{6} \\ \frac{49}{6} & -\frac{37}{6} & \frac{23}{6} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} -14 \\ 8 \\ 44 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix}$$

Solution: $(-3, 5, 0)$

91. $\begin{cases} x + 2y = -1 \\ 3x + 4y = -5 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 \\ -5 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Solution: $(-3, 1)$

92. $\begin{cases} x + 3y = 23 \\ -6x + 2y = -18 \end{cases}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -6 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 23 \\ -18 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.15 \\ 0.3 & 0.05 \end{bmatrix} \begin{bmatrix} 23 \\ -18 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$x = 5, y = 6$

Solution: $(5, 6)$

93. $\begin{cases} -3x - 3y - 4z = 2 \\ y + z = -1 \\ 4x + 3y + 4z = -1 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 1 \\ 4 & 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 4 & 4 & 3 \\ -4 & -3 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Solution: $(1, 1, -2)$

94. $\begin{cases} x - 3y - 2z = 8 \\ -2x + 7y + 3z = -19 \\ x - y - 3z = 3 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 \\ -2 & 7 & 3 \\ 1 & -1 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 8 \\ -19 \\ 3 \end{bmatrix} = \begin{bmatrix} -18 & -7 & 5 \\ -3 & -1 & 1 \\ -5 & -2 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ -19 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$x = 4, y = -2, z = 1$

Solution: $(4, -2, 1)$

95. $\begin{vmatrix} 8 & 5 \\ 2 & -4 \end{vmatrix} = 8(-4) - 5(2) = -42$

96. $\begin{vmatrix} -9 & 11 \\ 7 & -4 \end{vmatrix} = (-9)(-4) - (11)(7) = -41$

97. $\begin{vmatrix} 50 & -30 \\ 10 & 5 \end{vmatrix} = 50(5) - (-30)(10) = 550$

98. $\begin{vmatrix} 14 & -24 \\ 12 & -15 \end{vmatrix} = (14)(-15) - (-24)(12) = 78$

99. $\begin{bmatrix} 2 & -1 \\ 7 & 4 \end{bmatrix}$

(a) $M_{11} = 4$
$M_{12} = 7$
$M_{21} = -1$
$M_{22} = 2$

(b) $C_{11} = M_{11} = 4$
$C_{12} = -M_{12} = -7$
$C_{21} = -M_{21} = 1$
$C_{22} = M_{22} = 2$

100. $\begin{bmatrix} 3 & 6 \\ 5 & -4 \end{bmatrix}$

(a) $M_{11} = -4$
$M_{12} = 5$
$M_{21} = 6$
$M_{22} = 3$

(b) $C_{11} = M_{11} = -4$
$C_{12} = -M_{12} = -5$
$C_{21} = -M_{21} = -6$
$C_{22} = M_{22} = 3$

101. $\begin{bmatrix} 3 & 2 & -1 \\ -2 & 5 & 0 \\ 1 & 8 & 6 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 5 & 0 \\ 8 & 6 \end{vmatrix} = 30$

$M_{12} = \begin{vmatrix} -2 & 0 \\ 1 & 6 \end{vmatrix} = -12$

$M_{13} = \begin{vmatrix} -2 & 5 \\ 1 & 8 \end{vmatrix} = -21$

$M_{21} = \begin{vmatrix} 2 & -1 \\ 8 & 6 \end{vmatrix} = 20$

$M_{22} = \begin{vmatrix} 3 & -1 \\ 1 & 6 \end{vmatrix} = 19$

$M_{23} = \begin{vmatrix} 3 & 2 \\ 1 & 8 \end{vmatrix} = 22$

$M_{31} = \begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} = 5$

$M_{32} = \begin{vmatrix} 3 & -1 \\ -2 & 0 \end{vmatrix} = -2$

$M_{33} = \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} = 19$

(b) $C_{11} = M_{11} = 30$
$C_{12} = -M_{12} = 12$
$C_{13} = M_{13} = -21$
$C_{21} = -M_{21} = -20$
$C_{22} = M_{22} = 19$
$C_{23} = -M_{23} = -22$
$C_{31} = M_{31} = 5$
$C_{32} = -M_{32} = 2$
$C_{33} = M_{33} = 19$

102. $\begin{bmatrix} 8 & 3 & 4 \\ 6 & 5 & -9 \\ -4 & 1 & 2 \end{bmatrix}$

(a) $M_{11} = \begin{vmatrix} 5 & -9 \\ 1 & 2 \end{vmatrix} = 19$

$M_{12} = \begin{vmatrix} 6 & -9 \\ -4 & 2 \end{vmatrix} = -24$

$M_{13} = \begin{vmatrix} 6 & 5 \\ -4 & 1 \end{vmatrix} = 26$

$M_{21} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 2$

$M_{22} = \begin{vmatrix} 8 & 4 \\ -4 & 2 \end{vmatrix} = 32$

$M_{23} = \begin{vmatrix} 8 & 3 \\ -4 & 1 \end{vmatrix} = 20$

$M_{31} = \begin{vmatrix} 3 & 4 \\ 5 & -9 \end{vmatrix} = -47$

$M_{32} = \begin{vmatrix} 8 & 4 \\ 6 & -9 \end{vmatrix} = -96$

$M_{33} = \begin{vmatrix} 8 & 3 \\ 6 & 5 \end{vmatrix} = 22$

(b) $C_{11} = M_{11} = 19$
$C_{12} = -M_{12} = 24$
$C_{13} = M_{13} = 26$
$C_{21} = -M_{21} = -2$
$C_{22} = M_{22} = 32$
$C_{23} = -M_{23} = -20$
$C_{31} = M_{31} = -47$
$C_{32} = -M_{32} = 96$
$C_{33} = M_{33} = 22$

103. Expand using Column 2.

$$\begin{vmatrix} -2 & 4 & 1 \\ -6 & 0 & 2 \\ 5 & 3 & 4 \end{vmatrix} = -4\begin{vmatrix} -6 & 2 \\ 5 & 4 \end{vmatrix} - 3\begin{vmatrix} -2 & 1 \\ -6 & 2 \end{vmatrix}$$

$$= -4(-34) - 3(2) = 130$$

104. Expand using Row 3.

$$\begin{vmatrix} 4 & 7 & -1 \\ 2 & -3 & 4 \\ -5 & 1 & -1 \end{vmatrix} = -5\begin{vmatrix} 7 & -1 \\ -3 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & -1 \\ 2 & 4 \end{vmatrix} - 1\begin{vmatrix} 4 & 7 \\ 2 & -3 \end{vmatrix}$$

$$= -5(25) - (18) - (-26) = -117$$

105. Expand along Row 1.

$$\begin{vmatrix} 3 & 0 & -4 & 0 \\ 0 & 8 & 1 & 2 \\ 6 & 1 & 8 & 2 \\ 0 & 3 & -4 & 1 \end{vmatrix} = 3\begin{vmatrix} 8 & 1 & 2 \\ 1 & 8 & 2 \\ 3 & -4 & 1 \end{vmatrix} + (-4)\begin{vmatrix} 0 & 8 & 2 \\ 6 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= 3[8(8 - (-8)) - 1(1 - 6) + 2(-4 - 24)] - 4[0 - 6(8 - 6) + 0]$$

$$= 3[128 + 5 - 56] - 4[-12]$$

$$= 279$$

106. Expand using Row 1, then use Row 3 of each 3×3 matrix.

$$\begin{vmatrix} -5 & 6 & 0 & 0 \\ 0 & 1 & -1 & 2 \\ -3 & 4 & -5 & 1 \\ 1 & 6 & 0 & 3 \end{vmatrix} = -5\begin{vmatrix} 1 & -1 & 2 \\ 4 & -5 & 1 \\ 6 & 0 & 3 \end{vmatrix} - 6\begin{vmatrix} 0 & -1 & 2 \\ -3 & -5 & 1 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= -5[6(-1 + 10) + 3(-5 + 4)] - 6[(-1 + 10) + 3(0 - 3)]$$

$$= -5(54 - 3) - 6(9 - 9)$$

$$= -255$$

107. $\begin{cases} 5x - 2y = 6 \\ -11x + 3y = -23 \end{cases}$

$$x = \frac{\begin{vmatrix} 6 & -2 \\ -23 & 3 \end{vmatrix}}{\begin{vmatrix} 5 & -2 \\ -11 & 3 \end{vmatrix}} = \frac{-28}{-7} = 4, \qquad y = \frac{\begin{vmatrix} 5 & 6 \\ -11 & -23 \end{vmatrix}}{\begin{vmatrix} 5 & -2 \\ -11 & 3 \end{vmatrix}} = \frac{-49}{-7} = 7$$

Solution: $(4, 7)$

108. $\begin{cases} 3x + 8y = -7 \\ 9x - 5y = 37 \end{cases}$

$$x = \frac{\begin{vmatrix} -7 & 8 \\ 37 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \\ 9 & -5 \end{vmatrix}} = \frac{-261}{-87} = 3, \qquad y = \frac{\begin{vmatrix} 3 & -7 \\ 9 & 37 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \\ 9 & -5 \end{vmatrix}} = \frac{174}{-87} = -2$$

Solution: $(3, -2)$

109. $\begin{cases} -2x + 3y - 5z = -11 \\ 4x - y + z = -3 \\ -x - 4y + 6z = 15 \end{cases}$

$$D = \begin{vmatrix} -2 & 3 & -5 \\ 4 & -1 & 1 \\ -1 & -4 & 6 \end{vmatrix} = -2(-1)^2\begin{vmatrix} -1 & 1 \\ -4 & 6 \end{vmatrix} + 4(-1)^3\begin{vmatrix} 3 & -5 \\ -4 & 6 \end{vmatrix} - 1(-1)^4\begin{vmatrix} 3 & -5 \\ -1 & 1 \end{vmatrix}$$

$$= -2(-2) - 4(-2) - (-2) = 14$$

$$x = \frac{\begin{vmatrix} -11 & 3 & -5 \\ -3 & -1 & 1 \\ 15 & -4 & 6 \end{vmatrix}}{14} = \frac{-11(-1)^2\begin{vmatrix} -1 & 1 \\ -4 & 6 \end{vmatrix} - 3(-1)^3\begin{vmatrix} 3 & -5 \\ -4 & 6 \end{vmatrix} + 15(-1)^4\begin{vmatrix} 3 & -5 \\ -1 & 1 \end{vmatrix}}{14}$$

$$= \frac{-11(-2) + 3(-2) + 15(-2)}{14} = \frac{-14}{14} = -1$$

$$y = \frac{\begin{vmatrix} -2 & -11 & -5 \\ 4 & -3 & 1 \\ -1 & 15 & 6 \end{vmatrix}}{14} = \frac{-2(-1)^2\begin{vmatrix} -3 & 1 \\ 15 & 6 \end{vmatrix} + 4(-1)^3\begin{vmatrix} -11 & -5 \\ 15 & 6 \end{vmatrix} - 1(-1)^4\begin{vmatrix} -11 & -5 \\ -3 & 1 \end{vmatrix}}{14}$$

$$= \frac{-2(-33) - 4(9) - 1(-26)}{14} = \frac{56}{14} = 4$$

$$z = \frac{\begin{vmatrix} -2 & 3 & -11 \\ 4 & -1 & -3 \\ -1 & -4 & 15 \end{vmatrix}}{14} = \frac{-2(-1)^2\begin{vmatrix} -1 & -3 \\ -4 & 15 \end{vmatrix} + 4(-1)^3\begin{vmatrix} 3 & -11 \\ -4 & 15 \end{vmatrix} - 1(-1)^4\begin{vmatrix} 3 & -11 \\ -1 & -3 \end{vmatrix}}{14}$$

$$= \frac{-2(-27) - 4(1) - 1(-20)}{14} = \frac{70}{14} = 5$$

Solution: $(-1, 4, 5)$

110. $\begin{cases} 5x - 2y + z = 15 \\ 3x - 3y - z = -7, \\ 2x - y - 7z = -3 \end{cases}$ $\quad D = \begin{vmatrix} 5 & -2 & 1 \\ 3 & -3 & -1 \\ 2 & -1 & -7 \end{vmatrix} = 65$

$$x = \frac{\begin{vmatrix} 15 & -2 & 1 \\ -7 & -3 & -1 \\ -3 & -1 & -7 \end{vmatrix}}{65} = \frac{390}{65} = 6, \quad y = \frac{\begin{vmatrix} 5 & 15 & 1 \\ 3 & -7 & -1 \\ 2 & -3 & -7 \end{vmatrix}}{65} = \frac{520}{65} = 8, \quad z = \frac{\begin{vmatrix} 5 & -2 & 15 \\ 3 & -3 & -7 \\ 2 & -1 & -3 \end{vmatrix}}{65} = \frac{65}{65} = 1$$

Solution: $(6, 8, 1)$

111. $(1, 0), (5, 0), (5, 8)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 8 & 1 \end{vmatrix} = \frac{1}{2}\left(1\begin{vmatrix} 0 & 1 \\ 8 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 0 \\ 5 & 8 \end{vmatrix}\right) = \frac{1}{2}(-8 + 40) = \frac{1}{2}(32) = 16 \text{ square units}$$

112. $(-4, 0), (4, 0), (0, 6)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -4 & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 6 & 1 \end{vmatrix} = \frac{1}{2}(48) = 24 \text{ square units}$$

113. $(1, -4), (-2, 3), (0, 5)$

$$\text{Area} = -\frac{1}{2}\begin{vmatrix} 1 & -4 & 1 \\ -2 & 3 & 1 \\ 0 & 5 & 1 \end{vmatrix}$$

$$= -\frac{1}{2}\left(-5\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -4 \\ -2 & 3 \end{vmatrix}\right)$$

$$= -\frac{1}{2}(-5(3) + (-5)) = 10 \text{ square units}$$

114. $\left(\frac{3}{2}, 1\right), \left(4, -\frac{1}{2}\right), (4, 2)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} \frac{3}{2} & 1 & 1 \\ 4 & -\frac{1}{2} & 1 \\ 4 & 2 & 1 \end{vmatrix} = \frac{1}{2}\left(\frac{25}{4}\right) = \frac{25}{8} \text{ square units}$$

115. $(-1, 7), (3, -9), (-3, 15)$

$$\begin{vmatrix} -1 & 7 & 1 \\ 3 & -9 & 1 \\ -3 & 15 & 1 \end{vmatrix} = 0$$

The points are collinear.

116. Points: $(0, -5), (-2, -6), (8, -1)$

$$\begin{vmatrix} 0 & -5 & 1 \\ -2 & -6 & 1 \\ 8 & -1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & -6 \\ 8 & -1 \end{vmatrix} - \begin{vmatrix} 0 & -5 \\ 8 & -1 \end{vmatrix} + \begin{vmatrix} 0 & -5 \\ -2 & -6 \end{vmatrix}$$

$$= 50 - 40 - 10 = 0$$

The points are collinear.

117. $(-4, 0), (4, 4)$

$$\begin{vmatrix} x & y & 1 \\ -4 & 0 & 1 \\ 4 & 4 & 1 \end{vmatrix} = 0$$

$$1\begin{vmatrix} -4 & 0 \\ 4 & 4 \end{vmatrix} - 1\begin{vmatrix} x & y \\ 4 & 4 \end{vmatrix} + 1\begin{vmatrix} x & y \\ -4 & 0 \end{vmatrix} = 0$$

$$-16 - (4x - 4y) + 4y = 0$$

$$-4x + 8y - 16 = 0$$

$$x - 2y + 4 = 0$$

118. $(2, 5), (6, -1)$

$$\begin{vmatrix} x & y & 1 \\ 2 & 5 & 1 \\ 6 & -1 & 1 \end{vmatrix} = 0$$

$$6x + 4y - 32 = 0$$

$$3x + 2y - 16 = 0$$

119. $\left(-\frac{5}{2}, 3\right), \left(\frac{7}{2}, 1\right)$

$$\begin{vmatrix} x & y & 1 \\ -\frac{5}{2} & 3 & 1 \\ \frac{7}{2} & 1 & 1 \end{vmatrix} = 0$$

$$1\begin{vmatrix} -\frac{5}{2} & 3 \\ \frac{7}{2} & 1 \end{vmatrix} - 1\begin{vmatrix} x & y \\ \frac{7}{2} & 1 \end{vmatrix} + 1\begin{vmatrix} x & y \\ -\frac{5}{2} & 3 \end{vmatrix} = 0$$

$$-13 - \left(x - \tfrac{7}{2}y\right) + \left(3x + \tfrac{5}{2}y\right) = 0$$

$$2x + 6y - 13 = 0$$

120. $(-0.8, 0.2), (0.7, 3.2)$

$$\begin{vmatrix} x & y & 1 \\ -0.8 & 0.2 & 1 \\ 0.7 & 3.2 & 1 \end{vmatrix} = 0$$

$-3x + 1.5y - 2.7 = 0$ Multiply both sides by $-\frac{10}{3}$.

$10x - 5y + 9 = 0$

121. L O O K __ O U T __ B E L O W __
[12 15 15] [11 0 15] [21 20 0] [2 5 12] [15 23 0]

$$A = \begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix}$$

$$[12 \quad 15 \quad 15]\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix} = [-21 \quad 6 \quad 0]$$

$$[11 \quad 0 \quad 15]\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix} = [-68 \quad 8 \quad 45]$$

$$[21 \quad 20 \quad 0]\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix} = [102 \quad -42 \quad -60]$$

$$[2 \quad 5 \quad 12]\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix} = [-53 \quad 20 \quad 21]$$

$$[15 \quad 23 \quad 0]\begin{bmatrix} 2 & -2 & 0 \\ 3 & 0 & -3 \\ -6 & 2 & 3 \end{bmatrix} = [99 \quad -30 \quad -69]$$

Cryptogram: -21 6 0 -68 8 45 102
-42 -60 -53 20 21 99 -30 -69

122. R E T U R N __ T 0 __ B A S E __
[18 5 20] [21 18 14] [0 20 15] [0 2 1] [19 5 0]

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -6 & -6 & -2 \\ 3 & 2 & 1 \end{bmatrix}$$

$[18 \quad 5 \quad 20]A = [66 \quad 28 \quad 10]$

$[21 \quad 18 \quad 14]A = [-24 \quad -59 \quad -22]$

$[0 \quad 20 \quad 15]A = [-75 \quad -90 \quad -25]$

$[0 \quad 2 \quad 1]A = [-9 \quad -10 \quad -3]$

$[19 \quad 5 \quad 0]A = [8 \quad -11 \quad -10]$

Cryptogram: 66 28 10 -24 -59 -22 -75 -90 -25 -9 -10 -3 8 -11 -10

123. $A^{-1} = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$$\begin{bmatrix} -5 & 11 & -2 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 19 & 5 & 5 \end{bmatrix} \quad \text{S} \quad \text{E} \quad \text{E}$$

$$\begin{bmatrix} 370 & -265 & 225 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 25 & 15 \end{bmatrix} \quad \_\_ \quad \text{Y} \quad \text{O}$$

$$\begin{bmatrix} -57 & 48 & -33 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 6 \end{bmatrix} \quad \text{U} \quad \_\_ \quad \text{F}$$

$$\begin{bmatrix} 32 & -15 & 20 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 18 & 9 & 4 \end{bmatrix} \quad \text{R} \quad \text{I} \quad \text{D}$$

$$\begin{bmatrix} 245 & -171 & 147 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 25 & 0 \end{bmatrix} \quad \text{A} \quad \text{Y} \quad \_\_$$

Message: SEE YOU FRIDAY

124. $A^{-1} = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

$$\begin{bmatrix} 145 & -105 & 92 \\ 264 & -188 & 160 \\ 23 & -16 & 15 \\ 129 & -84 & 78 \\ -9 & 8 & -5 \\ 159 & -118 & 100 \\ 219 & -152 & 133 \\ 370 & -265 & 225 \\ -105 & 84 & -63 \end{bmatrix}\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 13 & 1 & 25 \\ 0 & 20 & 8 \\ 5 & 0 & 6 \\ 15 & 18 & 3 \\ 5 & 0 & 2 \\ 5 & 0 & 23 \\ 9 & 20 & 8 \\ 0 & 25 & 15 \\ 21 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \text{M} & \text{A} & \text{Y} \\ \_\_ & \text{T} & \text{H} \\ \text{E} & \_\_ & \text{F} \\ \text{O} & \text{R} & \text{C} \\ \text{E} & \_\_ & \text{B} \\ \text{E} & \_\_ & \text{W} \\ \text{I} & \text{T} & \text{H} \\ \_\_ & \text{Y} & \text{O} \\ \text{U} & \_\_ & \_\_ \end{matrix}$$

Message: MAY THE FORCE BE WITH YOU

125. False. The matrix must be square.

126. True. Expand along Row 3.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+c_1 & a_{32}+c_2 & a_{33}+c_3 \end{vmatrix} = (a_{31}+c_1)\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - (a_{32}+c_2)\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + (a_{33}+c_3)\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - a_{32}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + a_{33}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$+ c_1\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} - c_2\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} + c_3\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Note: Expand each of these matrices along Row 3 to see the previous step.

127. The matrix must be square and its determinant nonzero to have an inverse.

128. If A is a square matrix, the cofactor C_{ij} of the entry a_{ij} is $(-1)^{i+j}M_{ij}$, where M_{ij} is the determinant obtained by deleting the ith row and jth column of A. The determinant of A is the sum of the entries of any row or column of A multiplied by their respective cofactors.

129. No. Each matrix is in row-echelon form, but the third matrix cannot be achieved from the first or second matrix with elementary row operations. Also, the first two matrices describe a system of equations with one solution. The third matrix describes a system with infinitely many solutions.

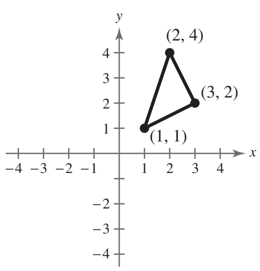
130. The part of the matrix corresponding to the coefficients of the system reduces to a matrix in which the number of rows with nonzero entries is the same as the number of variables.

131.
$$\begin{vmatrix} 2-\lambda & 5 \\ 3 & -8-\lambda \end{vmatrix} = 0$$
$$(2-\lambda)(-8-\lambda) - 15 = 0$$
$$-16 + 6\lambda + \lambda^2 - 15 = 0$$
$$\lambda^2 + 6\lambda - 31 = 0$$
$$\lambda = \frac{-6 \pm \sqrt{36 - 4(-31)}}{2}$$
$$\lambda = -3 \pm 2\sqrt{10}$$

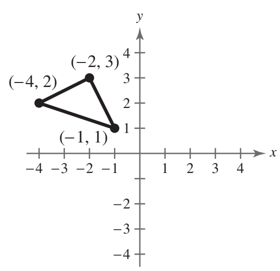
Problem Solving for Chapter 8

1. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad T = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$

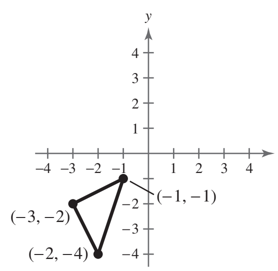
(a) $AT = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix}$ $\quad AAT = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -4 & -2 \end{bmatrix}$



Original Triangle



AT Triangle



AAT Triangle

The transformation A interchanges the x and y coordinates and then takes the negative of the x coordinate. A represents a counterclockwise rotation by 90°.

(b) $A^{-1}(AAT) = (A^{-1}A)(AT) = (I)(AT) = AT$

$A^{-1}(AT) = (A^{-1}A)T = IT = T$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

A^{-1} represents a clockwise rotation by 90°.

2. (a)

2000

$$\begin{array}{l} \begin{array}{ccc} 0\text{–}17 & 18\text{–}64 & 65+ \end{array} \\ \begin{bmatrix} 4.64\% & 11.79\% & 2.62\% \\ 5.91\% & 14.03\% & 2.94\% \\ 9.09\% & 22.11\% & 4.42\% \\ 1.75\% & 3.98\% & 0.72\% \\ 4.30\% & 9.96\% & 1.74\% \end{bmatrix} \end{array} \begin{array}{l} \text{Northeast} \\ \text{Midwest} \\ \text{South} \\ \text{Mountain} \\ \text{Pacific} \end{array}$$

2015

$$\begin{array}{l} \begin{array}{ccc} 0\text{–}17 & 18\text{–}64 & 65+ \end{array} \\ \begin{bmatrix} 4.06\% & 10.99\% & 2.63\% \\ 5.12\% & 13.23\% & 3.26\% \\ 8.36\% & 22.25\% & 5.63\% \\ 1.69\% & 4.07\% & 1.05\% \\ 4.81\% & 10.74\% & 2.12\% \end{bmatrix} \end{array} \begin{array}{l} \text{Northeast} \\ \text{Midwest} \\ \text{South} \\ \text{Mountain} \\ \text{Pacific} \end{array}$$

(b) Change in Percent of Population from 2000 to 2015

$$\begin{array}{l} \begin{array}{ccc} 0\text{–}17 & 18\text{–}64 & 65+ \end{array} \\ \begin{bmatrix} -0.58\% & -0.80\% & 0.01\% \\ -0.79\% & -0.80\% & 0.32\% \\ -0.73\% & 0.14\% & 1.21\% \\ -0.06\% & 0.09\% & 0.33\% \\ 0.51\% & 0.78\% & 0.38\% \end{bmatrix} \end{array} \begin{array}{l} \text{Northeast} \\ \text{Midwest} \\ \text{South} \\ \text{Mountain} \\ \text{Pacific} \end{array}$$

(c) All regions show growth in the 65+ age bracket, especially the South. The South, Mountain and Pacific regions show growth in the 18–64 age bracket. Only the Pacific region shows growth in the 0–17 age bracket.

3. (a) $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A$

A *is* idempotent.

(b) $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq A$

A is *not* idempotent.

(c) $A^2 = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq A$

A is *not* idempotent.

(d) $A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \neq A$

A is *not* idempotent.

4. $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

(a)
$$\begin{aligned} A^2 - 2A + 5I &= \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \end{aligned}$$

(b)
$$A^{-1} = \frac{1}{(1) - (-4)}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$\frac{1}{5}(2I - A) = \frac{1}{5}\left[\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\right] = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

Thus, $A^{-1} = \frac{1}{5}(2I - A)$.

(c)
$$\begin{aligned} A^2 - 2A + 5I &= 0 \\ A^2 - 2A &= -5I \\ (A - 2I)A &= -5I \\ -\frac{1}{5}(A - 2I)A &= I \\ \frac{1}{5}(2I - A)A &= I \end{aligned}$$

Thus, $A^{-1} = \frac{1}{5}(2I - A)$.

5. (a) $\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix} \begin{bmatrix} 25{,}000 \\ 30{,}000 \\ 45{,}000 \end{bmatrix} = \begin{bmatrix} 28{,}750 \\ 35{,}750 \\ 35{,}500 \end{bmatrix}$

Gold Cable Company: 28,750 households

Galaxy Cable Company: 35,750 households

Nonsubscribers: 35,500 households

(b) $\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix} \begin{bmatrix} 28{,}750 \\ 35{,}750 \\ 35{,}500 \end{bmatrix} \approx \begin{bmatrix} 30{,}813 \\ 39{,}675 \\ 29{,}513 \end{bmatrix}$

Gold Cable Company: 30,813 households

Galaxy Cable Company: 39,675 households

Nonsubscribers: 29,513 households

(c) $\begin{bmatrix} 0.70 & 0.15 & 0.15 \\ 0.20 & 0.80 & 0.15 \\ 0.10 & 0.05 & 0.70 \end{bmatrix} \begin{bmatrix} 30{,}812.5 \\ 39{,}675 \\ 29{,}512.5 \end{bmatrix} \approx \begin{bmatrix} 31{,}947 \\ 42{,}329 \\ 25{,}724 \end{bmatrix}$

Gold Cable Company: 31,947 households

Galaxy Cable Company: 42,329 households

Nonsubscribers: 25,724 households

(d) Both cable companies are increasing the number of subscribers, while the number of nonsubscribers is decreasing each year.

6. $A = \begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix} \Rightarrow A^{-1} = \dfrac{1}{-9 + 2x} \begin{bmatrix} -3 & -x \\ 2 & 3 \end{bmatrix}$

If $A = A^{-1}$, then $\begin{bmatrix} \dfrac{-3}{-9 + 2x} & \dfrac{-x}{-9 + 2x} \\ \dfrac{2}{-9 + 2x} & \dfrac{3}{-9 + 2x} \end{bmatrix} = \begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix}$.

Equating the first entry in Row 1 yields $\dfrac{-3}{-9 + 2x} = 3 \Rightarrow -3 = -27 + 6x \Rightarrow x = 4$.

Now check $x = 4$ in the other entries:

$\dfrac{-4}{-9 + 2(4)} = 4$ ✓

$\dfrac{2}{-9 + 2(4)} = -2$ ✓

$\dfrac{3}{-9 + 2(4)} = -3$ ✓

Thus, $x = 4$.

7. If $A = \begin{bmatrix} 4 & x \\ -2 & -3 \end{bmatrix}$ is singular then

$ad - bc = -12 + 2x = 0$.

Thus, $x = 6$.

8. From Exercise 3 we have the singular matrix

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ where $A^2 = A$.

Also, $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ has this property.

9. $(a - b)(b - c)(c - a) = -a^2b + a^2c + ab^2 - ac^2 - b^2c + bc^2$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - \begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + \begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} = bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b$$

Thus, $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$.

10. $(a - b)(b - c)(c - a)(a + b + c) = -a^3b + a^3c + ab^3 - ac^3 - b^3c + bc^3$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \begin{vmatrix} b & c \\ b^3 & c^3 \end{vmatrix} - \begin{vmatrix} a & c \\ a^3 & c^3 \end{vmatrix} + \begin{vmatrix} a & b \\ a^3 & b^3 \end{vmatrix} = bc^3 - b^3c - ac^3 + a^3c + ab^3 - a^3b$$

Thus, $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c).$

11. $\begin{vmatrix} x & 0 & c \\ -1 & x & b \\ 0 & -1 & a \end{vmatrix} = x\begin{vmatrix} x & b \\ -1 & a \end{vmatrix} + c\begin{vmatrix} -1 & x \\ 0 & -1 \end{vmatrix} = x(ax + b) + c(1 - 0) = ax^2 + bx + c$

12. $\begin{vmatrix} x & 0 & 0 & d \\ -1 & x & 0 & c \\ 0 & -1 & x & b \\ 0 & 0 & -1 & a \end{vmatrix} = x\begin{vmatrix} x & 0 & c \\ -1 & x & b \\ 0 & -1 & a \end{vmatrix} - d\begin{vmatrix} -1 & x & 0 \\ 0 & -1 & x \\ 0 & 0 & -1 \end{vmatrix} = \underbrace{x(ax^2 + bx + c)}_{\text{From Exercise 11}} - d\left(- \begin{vmatrix} -1 & x \\ 0 & -1 \end{vmatrix}\right)$

$= ax^3 + bx^2 + cx + d$

13. $4S + 4N \qquad = 184$

$S \qquad + 6F = 146$

$2N + 4F = 104$

$D = \begin{vmatrix} 4 & 4 & 0 \\ 1 & 0 & 6 \\ 0 & 2 & 4 \end{vmatrix} = -64$

$S = \dfrac{\begin{vmatrix} 184 & 4 & 0 \\ 146 & 0 & 6 \\ 104 & 2 & 4 \end{vmatrix}}{-64} = \dfrac{-2048}{-64} = 32$

$N = \dfrac{\begin{vmatrix} 4 & 184 & 0 \\ 1 & 146 & 6 \\ 0 & 104 & 4 \end{vmatrix}}{-64} = \dfrac{-896}{-64} = 14$

$F = \dfrac{\begin{vmatrix} 4 & 4 & 184 \\ 1 & 0 & 146 \\ 0 & 2 & 104 \end{vmatrix}}{-64} = \dfrac{-1216}{-64} = 19$

Element	*Atomic mass*
Sulfur	32
Nitrogen	14
Fluoride	19

14. Let x = cost of a transformer, y = cost per foot of wire, z = cost of a light.

$x + 25y + 5z = 20$

$x + 50y + 15z = 35$

$x + 100y + 20z = 50$

$$\left[\begin{array}{ccc:c} 1 & 25 & 5 & 20 \\ 1 & 50 & 15 & 35 \\ 1 & 100 & 20 & 50 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0.2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

By using the matrix capabilities of a graphing calculator to reduce the augmented matrix to reduced row-echelon form, we have the following costs:

Transformer	\$10.00
Foot of wire	\$ 0.20
Light	\$ 1.00

15. $A = \begin{bmatrix} -1 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 1 & -1 \end{bmatrix}$

$A^T = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}, \qquad B^T = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}, \qquad (AB)^T = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$

$$B^TA^T = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$$

Thus, $(AB)^T = B^TA^T$.

16. $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 1 & -3 \\ 1 & -1 & 4 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{6}{11} & \frac{4}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{5}{11} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{3}{11} \end{bmatrix}$

$$\begin{bmatrix} 23 & 13 & -34 \\ 31 & -34 & 63 \\ 25 & -17 & 61 \\ 24 & 14 & -37 \\ 41 & -17 & -8 \\ 20 & -29 & 40 \\ 38 & -56 & 116 \\ 13 & -11 & 1 \\ 22 & -3 & -6 \\ 41 & -53 & 85 \\ 28 & -32 & 16 \end{bmatrix}\begin{bmatrix} \frac{1}{11} & \frac{6}{11} & \frac{4}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{5}{11} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{3}{11} \end{bmatrix} = \begin{bmatrix} 0 & 18 & 5 \\ 13 & 5 & 13 \\ 2 & 5 & 18 \\ 0 & 19 & 5 \\ 16 & 20 & 5 \\ 13 & 2 & 5 \\ 18 & 0 & 20 \\ 8 & 5 & 0 \\ 5 & 12 & 5 \\ 22 & 5 & 14 \\ 20 & 8 & 0 \end{bmatrix}$$

0	18	5	13	5	13	2	5	18	0			
__	R	E	M	E	M	B	E	R	__			
19	5	16	20	5	13	2	5	18	0			
S	E	P	T	E	M	B	E	R	__			
20	8	5	0	5	12	5	22	5	14	20	8	0
T	H	E	__	E	L	E	V	E	N	T	H	__

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17. (a) $\begin{bmatrix} 45 & -35 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 10 & 15 \end{bmatrix}$

$\begin{bmatrix} 38 & -30 \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 8 & 14 \end{bmatrix}$

$45w - 35y = 10$

$45x - 35z = 15$

$38w - 30y = 8$

$38x - 30z = 14$

$\left.\begin{aligned} 45w - 35y &= 10 \\ 38w - 30y &= 8 \end{aligned}\right\} \Rightarrow w = 1, y = 1$

$\left.\begin{aligned} 45x - 35z &= 15 \\ 38x - 30z &= 14 \end{aligned}\right\} \Rightarrow x = -2, z = -3$

$A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 45 & -35 \\ 38 & -30 \\ 18 & -18 \\ 35 & -30 \\ 81 & -60 \\ 42 & -28 \\ 75 & -55 \\ 2 & -2 \\ 22 & -21 \\ 15 & -10 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 8 & 14 \\ 0 & 18 \\ 5 & 20 \\ 21 & 18 \\ 14 & 0 \\ 20 & 15 \\ 0 & 2 \\ 1 & 19 \\ 5 & 0 \end{bmatrix} \begin{matrix} \text{J} & \text{O} \\ \text{H} & \text{N} \\ \text{—} & \text{R} \\ \text{E} & \text{T} \\ \text{U} & \text{R} \\ \text{N} & \text{—} \\ \text{T} & \text{O} \\ \text{—} & \text{B} \\ \text{A} & \text{S} \\ \text{E} & \text{—} \end{matrix}$

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18. $A = \begin{bmatrix} 6 & 4 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{16} & -\frac{7}{16} & \frac{5}{8} \\ \frac{3}{16} & \frac{11}{16} & -\frac{9}{8} \\ -\frac{1}{8} & -\frac{1}{8} & \frac{3}{4} \end{bmatrix}$

$|A| = 16$ and $|A^{-1}| = \dfrac{1}{16}$

Conjecture: $|A^{-1}| = \dfrac{1}{|A|}$

19. Let $A = \begin{bmatrix} 3 & -3 \\ 5 & -5 \end{bmatrix}$, then $|A| = 0$.

Let $A = \begin{bmatrix} 2 & 4 & -6 \\ -3 & 1 & 2 \\ 5 & -8 & 3 \end{bmatrix}$, then $|A| = 0$.

Let $A = \begin{bmatrix} 3 & -7 & 5 & -1 \\ -6 & 4 & 0 & 2 \\ 5 & 8 & -6 & -7 \\ 9 & 11 & -4 & -16 \end{bmatrix}$, then $|A| = 0$.

Conjecture: If A is an $n \times n$ matrix, each of whose rows add up to zero, then $|A| = 0$.

20. (a) Answers will vary.

$A = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$, $\quad B = \begin{bmatrix} 0 & 4 & -1 \\ 0 & 0 & 7 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $A^2 = 0$ so $A^n = 0$ for n an integer ≥ 2.

$B^2 = \begin{bmatrix} 0 & 0 & 28 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$B^3 = 0$ so $B^n = 0$ for n an integer ≥ 3.

(c) $A^4 = 0$ if A is 4×4.

(d) Conjecture: If A is $n \times n$, then $A^n = 0$.

Chapter 8 Practice Test

1. Put the matrix in reduced row-echelon form.

$$\begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 9 \end{bmatrix}$$

For Exercises 2–4, use matrices to solve the system of equations.

2. $\begin{cases} 3x + 5y = 3 \\ 2x - y = -11 \end{cases}$

3. $\begin{cases} 2x + 3y = -3 \\ 3x + 2y = 8 \\ x + y = 1 \end{cases}$

4. $\begin{cases} x + 3z = -5 \\ 2x + y = 0 \\ 3x + y - z = 3 \end{cases}$

5. Multiply $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & -3 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 0 & -7 \\ -1 & 2 \end{bmatrix}$.

6. Given $A = \begin{bmatrix} 9 & 1 \\ -4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix}$, find $3A - 5B$.

7. Find $f(A)$.

$f(x) = x^2 - 7x + 8,\ A = \begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix}$

8. True or false:

$(A + B)(A + 3B) = A^2 + 4AB + 3B^2$ where A and B are matrices.

(Assume that A^2, AB, and B^2 exist.)

For Exercises 9–10, find the inverse of the matrix, if it exists.

9. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 6 & 5 \\ 6 & 10 & 8 \end{bmatrix}$

11. Use an inverse matrix to solve the systems.

(a) $x + 2y = 4$
$3x + 5y = 1$

(b) $x + 2y = 3$
$3x + 5y = -2$

For Exercises 12–14, find the determinant of the matrix.

12. $\begin{bmatrix} 6 & -1 \\ 3 & 4 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 3 & -1 \\ 5 & 9 & 0 \\ 6 & 2 & -5 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 4 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 3 & 5 & -1 & 1 \\ 2 & 0 & 6 & 1 \end{bmatrix}$

15. Evaluate $\begin{vmatrix} 6 & 4 & 3 & 0 & 6 \\ 0 & 5 & 1 & 4 & 8 \\ 0 & 0 & 2 & 7 & 3 \\ 0 & 0 & 0 & 9 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$.

16. Use a determinant to find the area of the triangle with vertices $(0, 7)$, $(5, 0)$, and $(3, 9)$.

17. Find the equation of the line through $(2, 7)$ and $(-1, 4)$.

For Exercises 18–20, use Cramer's Rule to find the indicated value.

18. Find x.

$$\begin{cases} 6x - 7y = 4 \\ 2x + 5y = 11 \end{cases}$$

19. Find z.

$$\begin{cases} 3x \phantom{{}-y} + z = 1 \\ \phantom{3x+{}} y + 4z = 3 \\ x - y \phantom{{}+4z} = 2 \end{cases}$$

20. Find y.

$$\begin{cases} 721.4x - 29.1y = 33.77 \\ 45.9x + 105.6y = 19.85 \end{cases}$$